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WATER ENTRY IMPACT SHOCK ON FLAT FACED CYLINDRICAL MISSILES

BY S VAN DENK,
C. W. SMITH

UNDERWATER SYSTEMS DEPARTMENT

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Impact shock dynamics, including shock force and impulse as functions of time, are described for a flat faced cylindrical missile striking a smooth water surface with the missile's axis either normal or oblique to the water's surface. Shock dynamics calculations were facilitated by the development of a computer program, IMPACT, the use of which is described and a code listing given.

FOREWORD

This study of water entry impact shock on flat faced cylindrical missiles was sponsored by Dr. Thomas Peirce of NAVSEA, Code 63R31, whose continued support of water-entry research has made this extension of prior work possible.

This work may be applied to the determination of the shock component of water entry impact force, as a function of time, on air dropped torpedos and mines with stabilizing flat noses. The program IMPACT lessens the need for the much more expensive water entry impact testing on prototypes or models.

Approved by:

William C. Jones

WILLIAM C. JONES, Acting Head
Surface ASW Division

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PREFACE

The junior author would like to express his appreciation to Mr. Charles W. Smith and Dr. John L. Baldwin of NSWC's Hydroballistic Facility, Code U23, for their direction and support, and in making this study available as a rotational assignment project.

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ILLUSTRATIONS

INTRODUCTION

This study is an extension of a paper by Dallas D. Laumbach[1] of water entry impact shock on flat faced cylindrical missiles with their axis normal to the water's surface. This extension includes shock producing oblique entry impacts as well as normal impacts. The results of this work are such that for the normal entry case they reduce to those of Laumbach's.

PROBLEM DESCRIPTION

The problem treated by this report is the determination of the response dynamics of a flat faced cylindrical missile striking a smooth water surface in a shock producing impact. The missile enters the water with its axis parallel to its velocity vector and either normal or oblique to the water's surface. The dynamic response of a missile to water entry impact shock can be described by the impact shock force acting on the missile's face as a function of water entry time and its total impulse. To determine these quantities it is necessary to find the shock pressure, resulting from impact, between the water and the missile's face. This requires consideration of shock wave propagation, compressibility of water in shock, deformation of the missile's face under shock pressure, water particle speeds, and the geometry of the relieving expansion wave. Only the force due to shock pressure is found; gravity, buoyancy, and drag forces were not treated. The general problem for oblique entry is intrinsically three dimensional with time varying boundary conditions, due to gross penetration, coupling the compressing water and deforming missile.

GEOMETRY OF IMPACT

Described below is both the geometry of impact defining the problem to be solved and a nomenclature of phrases which will be used in reference to this geometry throughout the report.

1) The missile is a circular cylinder with radius r and of semi-infinite length.

[1]Laumbach, D.D., Impact Shock on a Blunt-Nosed Missile Striking the Water with its Axis Normal to the Water Surface, Sandia Corp. SC-TM-64-987, Sept 1964.

2) The end of this cylinder which strikes the water is initially a plane surface normal to the missile's axis. This is called the missile's face and its undeformed configuration is referred to as the missile's rigid face.

3) The undisturbed water is a semi-infinite half space bounded by a plane called the water's surface. This undisturbed water has an initial density of ρ_i at a pressure of p_i, 1 atmosphere, and a temperature of 20°C.

4) The missile enters the water with a constant, and prescribed, velocity vector which is parallel to the missile's axis.

5) This velocity vector is prescribed by a speed V' and an orientation α, the angle between the missile's axis and a normal to the water's surface.

6) Water entry impact time, t, is the time since the missile first contacts the water's surface.

7) For oblique water entry, $\alpha>0$, the point which first touches the water's surface is called the initial contact point and its associated generator of the missile's cylinder is called the initial entry edge.

8) Also for oblique entry, the contact between the missile's face and the undisturbed water surface is, for $t>0$, a chord on the missile's face called the wetting line.

9) Note that for normal water entry, $\alpha=0$, the full missile face is in initial contact with the water's surface at $t=0$ and there is no wetting line.

10) The wetting line moves across the undisturbed water surface with the speed $V'/\sin\alpha$, called the surface disturbance speed.

11) This surface disturbance, caused by the moving wetting line, results in a discontinuous disturbance of the water particles, imparting a velocity, and raising the water's density and pressure to ρ_f and p_f respectively. This disturbance, a shock, propagates radially as rays from the instantaneous wetting line position with a speed a, the shock wave speed in water.

12) These water shock rays may form the loci of a plane surface which subtends the wetting line and moves through the water with speed a. This surface is the shock wave front.

13) The wetting line moves across the face of the missile with a speed $V=V'/\tan\alpha$, called the wetting line speed.

14) Water in a shocked state recovers its initial properties in an expansion which propagates through the shocked water as a wave, moving with speed c', forming the expansion wave front.

15) The contour of this expansion wave front on the missile's face is called the expansion wave profile and moves over the missile's face with a projected speed c in the rigid face.

16) The region of the missile's face between the wetting line and the expansion wave profile encompasses water in shock, deforming the missile's face with shock pressure p_f. This region is called the shock pressure region and its area the shock pressure area S.

17) The missile's entry speed and angle are such that its impact is shock producing. (This requires that the surface disturbance speed be greater than the water's shock wave speed, establishing the shock wave front at the wetting line, and further, that the wetting line speed is greater than the expansion wave profile speed, resulting in a shock pressure region.)

18) The plane which contains the missile's axis and is normal to the water's surface is called the plane of symmetry. This plane contains the initial contact point, bisects the wetting line, and is a symmetry plane to the expansion wave profile and shock pressure area.

19) The loading by the water shock pressure on the missile's face initiates a deformation of the face at the wetting line which propagates strain into the missile as a shock wave as the shock pressure region moves across the missile's face. This missile shock wave has an axial speed of a_m.

20) The water expansion wave, restoring the water to its initial pressure, unloads the missile's deforming face of its shock pressure. This unloading propagates its effects as a wave restoring the missile's material to its initial undisturbed state.

The geometry of missile impact for oblique entry is illustrated by Figure 1, which depicts the plane of symmetry and the axial projection of the missile's face.

PHYSICS OF IMPACT SHOCKS

The physics of shocks in water, as distinct from gasses, is treated in depth by Robert H. Cole, with regard to underwater explosions, in Chapters 2 and 4 of his work.[2] The impact shock response of solids is summarized in the first and second chapters of the reference work by J.A. Zukas, et al[3] which deals with the recent techniques of analysis, including numerical, and experimental impact dynamics. What follows is a brief sketch of the response physics of the water and missile to impact shock as it applies to the problem of this study.

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[2]Cole, R.H., Underwater Explosions (Princeton: Princeton University Press, 1948).

[3]Zukas, J.A., Nicholas, T., Swift, H.F., Greszczuk, L.B., and Curran, D.R., Impact Dynamics (New York: John Wiley & Sons, Inc., 1982).

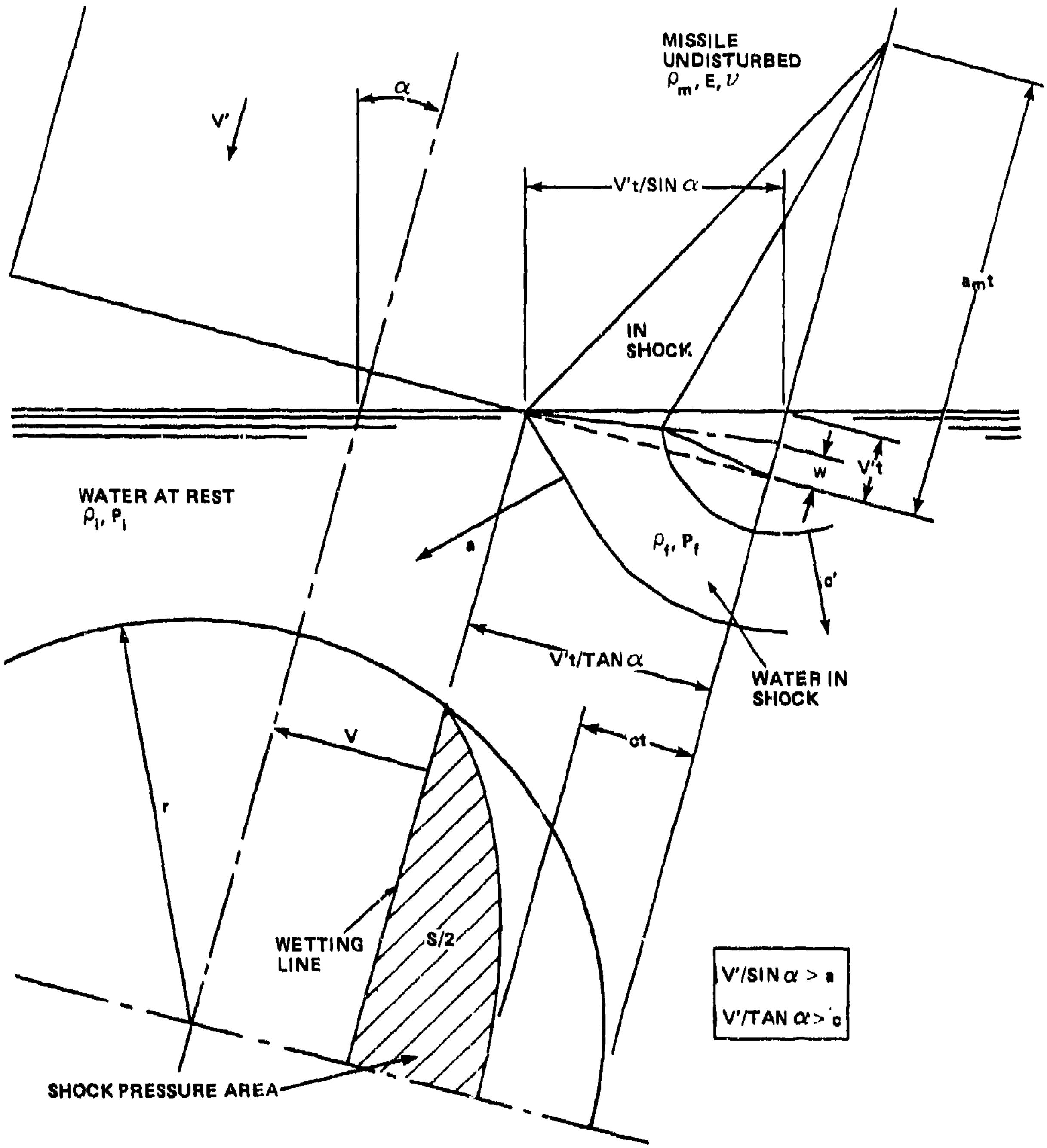

FIGURE 1. GEOMETRY OF A SHOCK PRODUCING OBLIQUE ENTRY IMPACT

Water Impact Response

Water entry by the missile imparts a disturbance speed to the water particles in contact with the missile's wetting face, initiating at the wetting line. These particles propagate their disturbance to the rest of the water as a compressive

shock wave, forming the discontinuity between disturbed and undisturbed regions of water. The rays of this shock wave are generated by the moving wetting line and propagate at the shock wave speed, a, from the position the wetting line had when
the ray was first generated. With the wetting line moving across the water's surface faster than the shock wave speed, the rays it generates form the loci of a plane, the shock wave front, which subtends the wetting line at an angle, the Mach angle, of less than 90°. With the missile then penetrating the shocked water, the water is compressed, resulting in an increase in density ρ_f and pressure p_f, the shock pressure.

Along the wetting circumference of the missile the water is free to expand and recover its initial state. This expansion propagates as a wave, the expansion wave, whose rays are generated at the instantaneous positions of the cusps of the wetting line on the missile's circumference. These rays form the loci of an expansion wave surface chasing the shock wave front. The water being compressed with the passage of the shock wave front is then later restored to its initial state with the passage of this expansion wave. Since water is unable to sustain tensile stresses this expansion wave is weak compared with the shock wave and moves through the water at the sonic speed c', which is less than the shock wave speed. This trailing of the expansion wave results in a shock pressure region contained between the shock wave front and the expansion wave surface. On the face of the missile the shock region is contained between the wetting line and the expansion wave profile on the missile's deforming face.

For the special case of normal water entry the face of the missile is in full contact with the water's surface at the instant of impact, generating a shock wave front parallel to the missile's face with a Mach angle of zero. Also, the expansion wave is generated simultaneously on the full circumference of the missile's face resulting in a circular expansion wave profile concentric with the circumference, and whose radius diminishes with speed c.

Missile Impact Response

For oblique water entry, shock pressure acting on the missile's face first loads the face at the wetting line as it moves across the face with speed V. This loading results in compressive axial deformation of the face and the propagation of a compressive shock initiating at the wetting line. This shock wave moves up the length of the missile as axial rays generated by the instantaneous position of the wetting line and travel with the wave speed a_m, forming an oblique wave front. The face continues to deform obliquely from the wetting line in the moving shock pressure region with an assumed constant deformation angle, ω, away from the rigid face position. This region of the face is then unloaded with the passage of the water's expansion wave profile, moving across the face at speed c, at which time compressive deformation ceases. This unloading also propagates its effects as an expansion wave which, since metals sustain full reversed tensile stresses, also travels at the shock wave speed a_m. This expansion wave in the missile, being generated by the moving water's expansion wave profile, creates a second oblique wave front in the missile restoring the missile's material to its initial unstrained state. This process of missile face deformation, shock wave propagation, and expansion wave unloading is illustrated, for the stages of water entry time T_e and impact shock duration time T_i, by Figure 2, which shows the missile in the plane of symmetry.

For the case of normal water entry impact, with the full face exposed to shock pressure at the instant of impact, the shock wave front and deformation surface become planes normal to the missile's axis, the deformation angle ω is zero. Also with the water's expansion wave moving in from the full circumference simultaneously, unloading the shock pressure region at speed c, the deformation profile in all axial planes becomes a truncated cone whose top progresses upward to form a full cone at shock duration time T_i.

BASIC ASSUMPTIONS FOR SOLUTION

Because a three dimensional elasticity/non-ideal compressible fluid dynamics model with variable boundary conditions is so complex, even for a finite difference approach on the resulting system of coupled nonlinear partial differential equations, a simplified, essentially one dimensional, "engineering" type of solution is sought. Such a solution requires a broad base of simplifying assumptions which reduce the problem to a soluble model without sacrificing the essential geometry and physics of the real phenomena. The basic assumptions employed for this solution are enumerated below:

1) The missile's water entry speed V' is constant during impact shock.

2) The missile's water entry angle α is also constant during impact shock.

3) Water shock is an adiabatic, inviscid, process with shock pressure and wave speed empirically related.

4) The impact shock pressure p_f is constant in time and uniform over the shock pressure region on the missile's face.

5) The missile's face under shock pressure deforms with a constant axial speed $\dot{w}$, initiated at the wetting line and ceasing at the expansion wave profile, resulting in a planar deformation surface spanning the shock pressure region. This deformation is described by an axial displacement w, which is the projection of the deformation plane to the initial entry edge and is measured along this edge from the undeformed face position as shown in Figure 1.

6) This deformation is small enough so as to result in a linear strain-displacement relation.

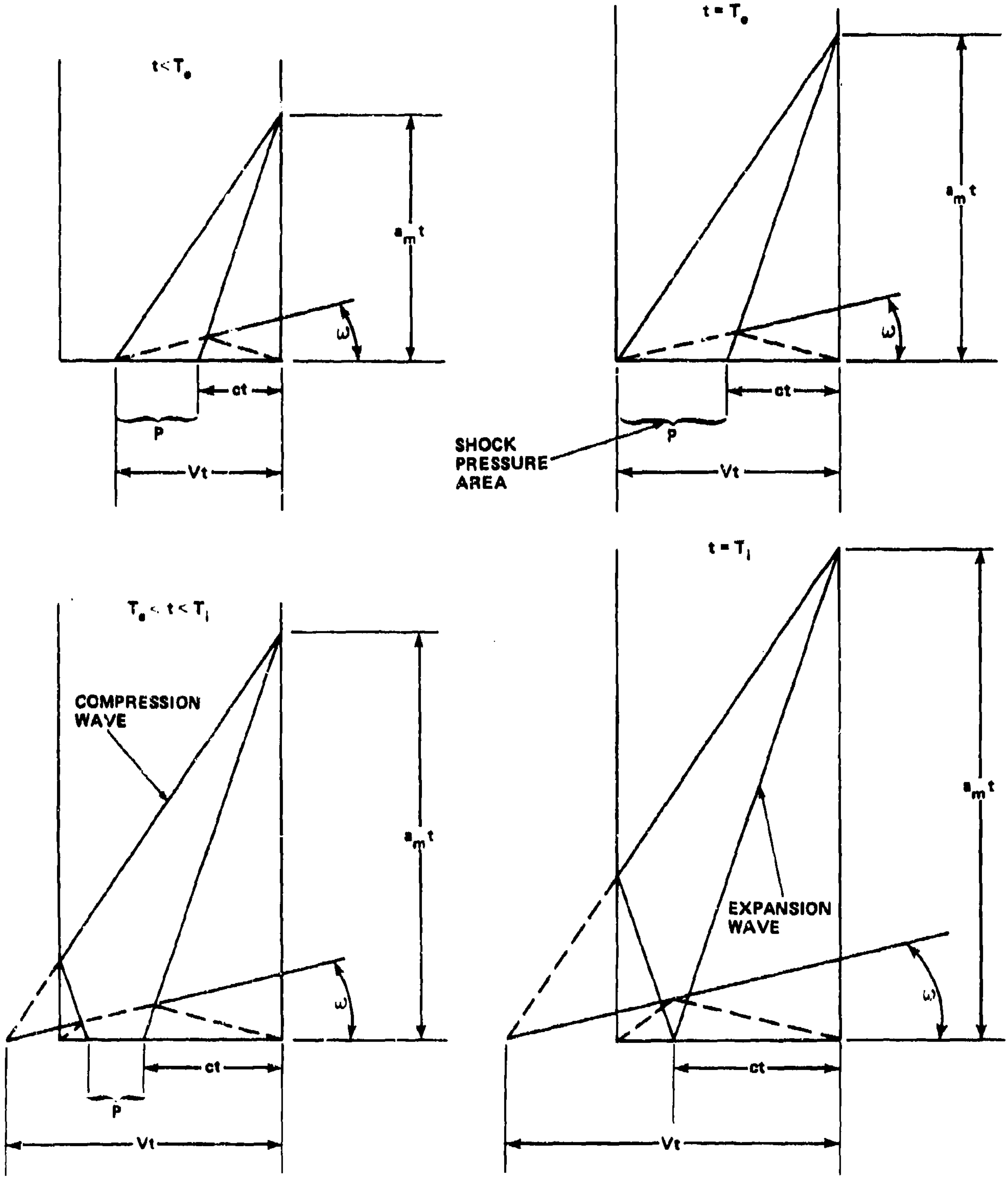

FIGURE 2. MISSILE FACE DEFORMATION UNDER SHOCK PRESSURE AREA

7) The missile is a homogeneous, isotropic, linearly elastic material of sufficient length so as to preclude shock wave interaction by reflections during impact shock. The missile is thus characterized by its initial or undisturbed properties of density ρ_m, Young's modulus E, and Poission's ratio ν.

8) The axial cylinder of the shock pressure region in the missile is constrained to have no lateral or transverse strains.

9) Both compression and expansion shock waves in the missile are one dimensional, moving axially with a speed a_m. The compression wave is initiated at the wetting line and the expansion wave is initiated at the water expansion wave profile.

10) The water expansion wave is sonic, moving with the speed of sound c' in water and restoring the water to its initial pressure p_i.

With the problem defined and the assumptions established a derivation of impact shock dynamics is now carried out, leading to the determination of shock force and impulse.

DERIVATION OF IMPACT SHOCK DYNAMICS

The equations which are the basis for determining the impact shock force and total shock impulse are now derived. This derivation is based upon the geometry, physics, and simplifying assumptions which were established in the Introduction. These allow an elementary or "engineering" solution of an otherwise complex problem in mechanics which it hopefully approximates.

MISSILE FACE DEFORMATION RESPONSE

As the missile enters the water, impact shock pressure produces a compressive deformation over the shock area of the wetted face. This axial displacement has been assumed to progress elastically with constant particle speed $\dot{w}(z,t)$, at the time of wetting, resulting in the linear deformation profiles in the plane of symmetry as depicted by Figure 2. Thus the displacement w of the deforming face extended to the initial entry edge, as shown in Figure 1, is

$$w(z=0,t) = \dot{w}(z=0,t)t.$$

The particle speed may be expressed as

$$\dot{w}(t,z) = \frac{\partial w}{\partial z}\,\frac{dz}{dt}$$

where

$$\frac{\partial w}{\partial z} = \lim_{\Delta z \to 0} \left[\left(w - \frac{\Delta w}{\Delta z} \cdot \Delta z\right) - w\right]/\Delta z$$

is the change in the length of Δz per unit length caused by the displacement field $w(z,t)$ acting along Δz, i.e. the axial strain

$$\epsilon_z = \frac{\partial w}{\partial z} ,$$

and

$$\frac{dz}{dt} = \lim_{\Delta t \to 0} \frac{\Delta z}{\Delta t}$$

is the distance Δz over which the particles have been displaced per unit time Δt, i.e. the speed with which the particles have been disturbed - the shock wave speed in the missile

$$a_m = \frac{dz}{dt} .$$

Therefore the displacement at the initial contact point becomes

$$w = \epsilon_z a_m t, \tag{1}$$

where $\epsilon_z = \epsilon_z(z=0,t)$ is the strain at the initial contact with t being the time since initial water contact by the missile.

Compressive Elastic Wave Speed

The missile has been assumed to respond elastically to impact. Furthermore, since the shock pressure area on the missile face is, except for normal impact at t=0, contained within the radius of its sides, and thus constrained by them, the elastic wave speed is derived as a one dimensional wave transmission along a uniform bar constrained to have no lateral or transverse displacements. This derivation is given by W. Johnson[4] and is developed below with the general situation illustrated by Figure 3.

[4]Johnson, W., Impact Strength of Materials (London: Edward Arnold Publishers, Ltd., 1972), pp. 13-14.

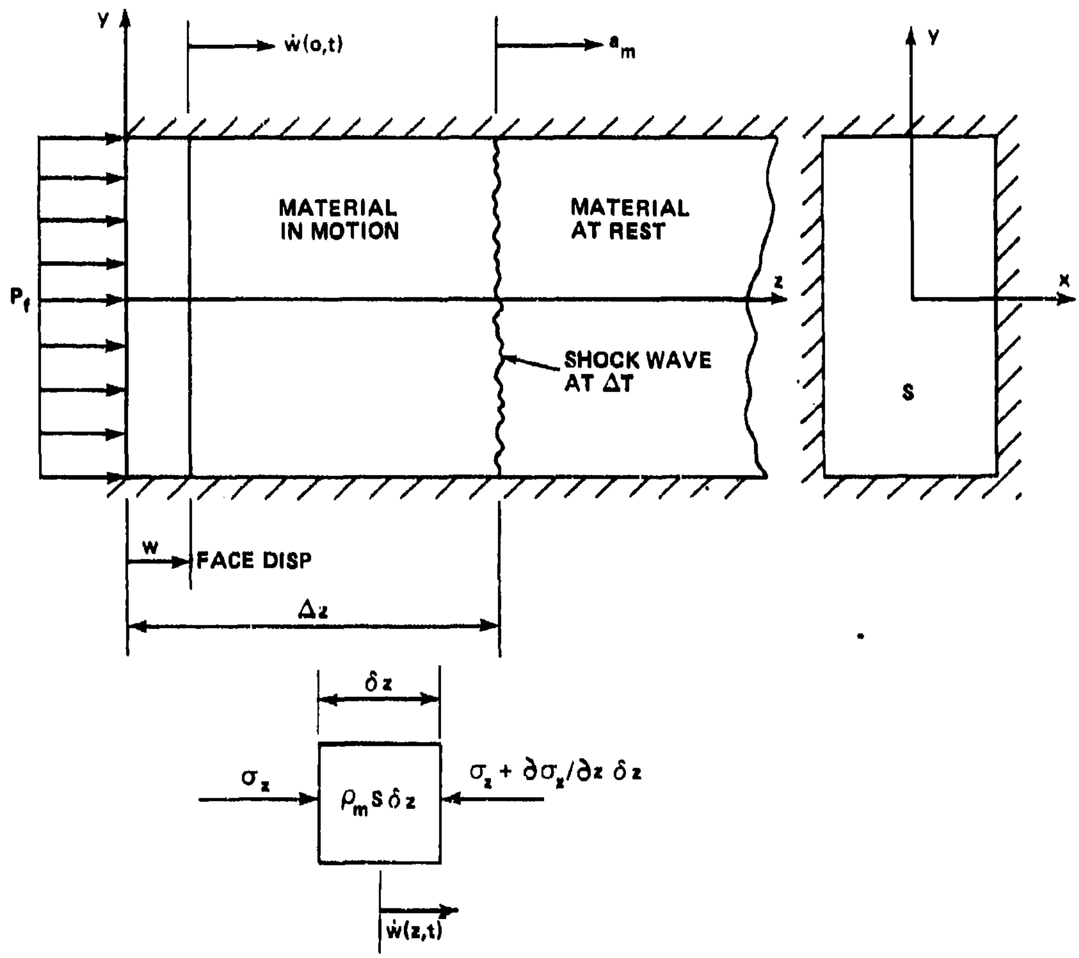

FIGURE 3. WAVE TRANSMISSION ALONG A UNIFORM BAR WITH NO LATERAL OR TRANSVERSE DISPLACEMENTS

The Hooke's Law stress-strain relation for linear elastic materials is, see for example Timoshenko and Goodier,[5]

$$\epsilon_x = \frac{1}{E}\,[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\epsilon_y = \frac{1}{E}\,[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\epsilon_z = \frac{1}{E}\,[\sigma_z - \nu(\sigma_x + \sigma_y)]\ ,$$

which with the zero lateral and transverse strain assumption

$$\epsilon_x = \epsilon_y = 0$$

becomes

$$0 = \sigma_x - \nu(\sigma_y + \sigma_z)$$

$$0 = \sigma_y - \nu(\sigma_x + \sigma_z)$$

$$\epsilon_z = \frac{1}{E}\,[\sigma_z - \nu(\sigma_x + \sigma_y)].$$

These require that

$$\sigma_x = \sigma_y = \frac{\nu}{1-\nu}\,\sigma_z \qquad \text{and}$$

$$\epsilon_z = \frac{\sigma_z}{E}\left(1 - \frac{2\nu^2}{1-\nu}\right). \tag{2}$$

[5]Timoshenko, S.P. and Goodier, J.N., Theory of Elasticity, 3rd. ed. (New York: McGraw-Hill Book Co., Inc., 1970), p. 8.

With the strain-displacement relation

$$\epsilon_z = \frac{\partial w}{\partial z} ,$$

a stress-displacement relation is found by substitution for ϵ_z as

$$\frac{\partial w}{\partial z} = \frac{\sigma_z}{E} \frac{(1+\nu)(1-2\nu)}{(1-\nu)}$$

or

$$\frac{\partial \sigma_z}{\partial z} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \frac{\partial^2 w}{\partial z^2} .$$

A balance of forces with momentum acting on an element of the material in shock, as shown in Figure 3, yields the equation of motion

$$\sigma_z S - (\sigma_z + \frac{\partial \sigma_z}{\partial z} \delta z) S = \rho_m (S \delta z) \frac{\partial^2 w}{\partial t^2}$$

or

$$\frac{\partial \sigma_z}{\partial z} = - \rho_m \frac{\partial^2 w}{\partial t^2} .$$

Substitution for stress σ_z from the previously determined stress-displacement relation gives the wave equation

$$\frac{\partial^2 w}{\partial t^2} = \frac{-E}{\rho_m} \frac{(1-\nu)}{(1+\nu)(1-2\nu)} \frac{\partial^2 w}{\partial z^2} .$$

Which implies an elastic wave speed in the missile of the form

$$a_m = \sqrt{\frac{E}{\rho_m} \frac{(1-\nu)}{(1+\nu)(1-2\nu)}} \ , \tag{3}$$

since

$$\frac{\partial^2 w}{\partial t^2} = -a_m^2 \frac{\partial^2 w}{\partial z^2}$$

has the general solution

$$w(z,t) = f(z-a_m t) + g(z+a_m t) \ ,$$

which are waves with profile forms f(z) and g(z) translating along the z-axis to the right and left respectively with wave speed a_m.

Axial Strain Under Shock Pressure

On the shock pressurized face of the missile the axial stress is the shock pressure

$$\sigma_z(0,t) = -p_f \ .$$

Substituting this into Equation (2) yields the strain at the initial contact point

$$\epsilon_z(0,t) = -\frac{p_f}{E} \cdot \frac{(1+\nu)(1-2\nu)}{(1-\nu)} \ .$$

Then introducing the wave speed from Equation (3) gives this strain-shock pressure relation the form

$$\epsilon_z = \frac{-p_f}{\rho_m \, a_m^2} \ . \tag{4}$$

Deformation Angle from a Rigid Face

With the initial water entry contact point displacement found the angle at which the face deforms from a rigid one in the plane of symmetry may now be determined. The geometry of this deformation is illustrated by Figure 4. As can be seen from this figure the deformation angle is

$$\tan \omega = \frac{\epsilon_z a_m t}{\frac{V' t}{\tan \alpha}}$$

or

$$\omega = \mathrm{Tan}^{-1}\left(\frac{\epsilon_z a_m \tan \alpha}{V'}\right). \tag{5}$$

WATER PARTICLE VELOCITY

The development of water shock dynamics requires a knowledge of the water particle velocity in the shock region, particularly that component normal to the shock wave front. The first step is to determine the resultant water particle velocity in this region relative to the water at rest, or equivalently, to the initial water entry contact point.

Resultant Water Particle Velocity

As the missile enters the water its face imparts motion to the water particles in the shocked region which move with the speed of the deforming face and in the direction normal to it. This deforming face speed relative to the fixed entry point is the entry speed of the missile less the deformation speed of the missile's face relative to the missile, or $\dot{w}$ of Equation (1). The geometry of these velocity vectors acting on a water particle in the shocked region is depicted by Figure 5. The component of this difference in the direction normal to the shock deformed face plane is the resultant water particle velocity in the shock pressure region,

$$u = (V' - \epsilon_z a_m) \cos \omega \tag{6}$$

Mach Angle of Water Shock Wave Front

With the wetting line moving across the surface of the water at a speed V, the surface disturbance speed, greater than the shock wave speed in water a shock

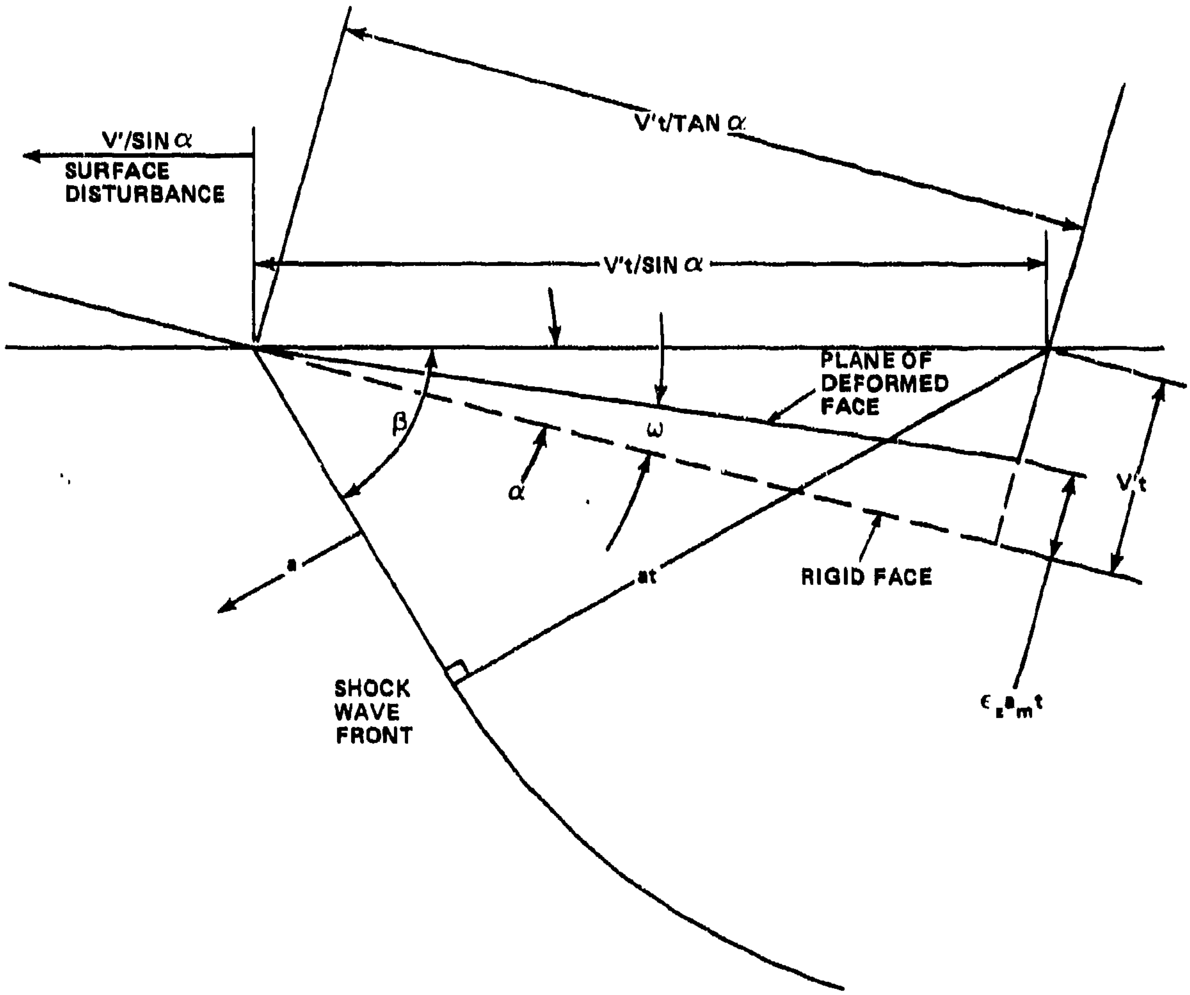

FIGURE 4. MISSILE FACE DEFORMATION AND THE WATER SHOCK WAVE FRONT

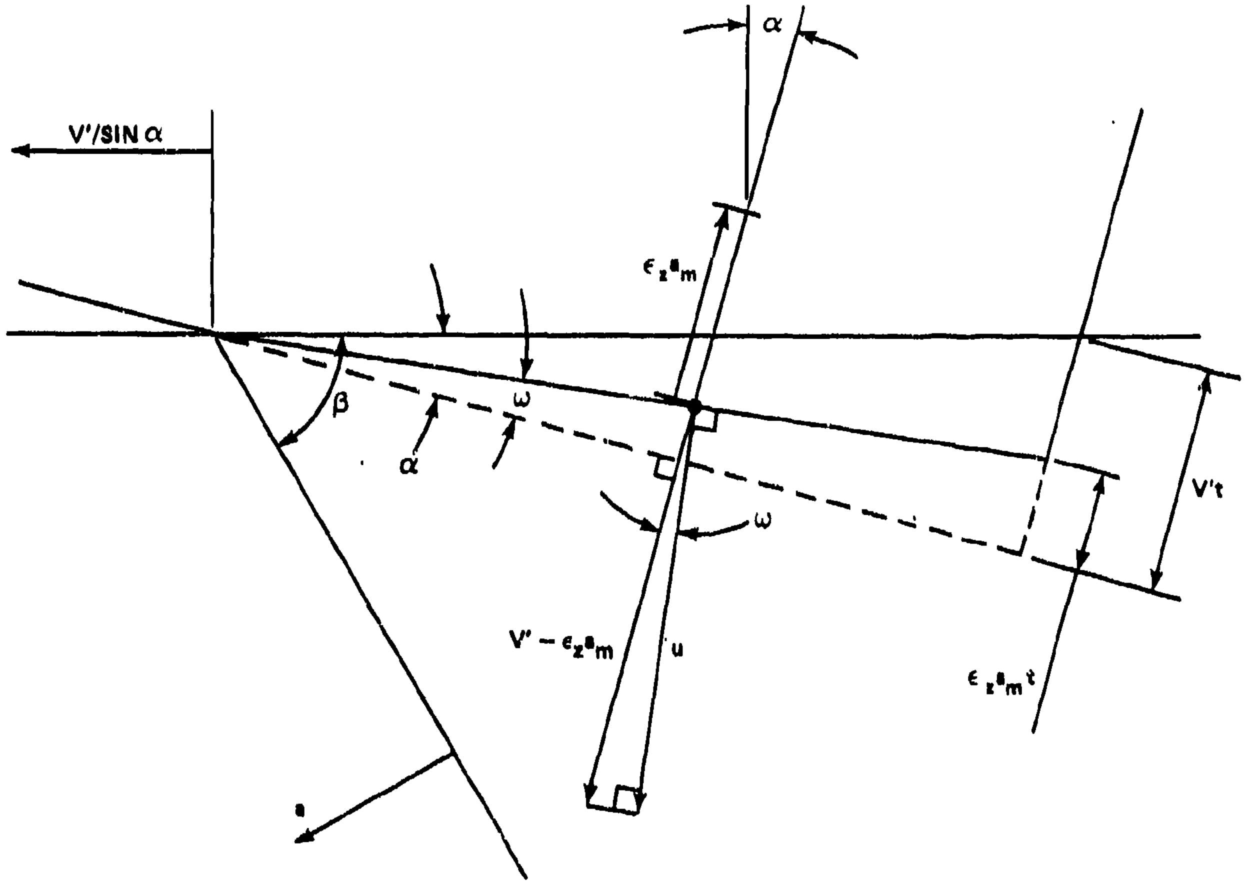

FIGURE 5. WATER PARTICLE VELOCITY IN THE SHOCKED REGION

wave front is established from the wetting line. The angle at which this wave front subtends the undisturbed surface is the Mach angle. The geometry is illustrated in Figure 4, from which

$$\sin \beta = \frac{at}{\frac{V't}{\sin \alpha}}$$

or

$$\beta = \sin^{-1}\left(\frac{a \sin \alpha}{V'}\right) . \tag{7}$$

For a Mach angle, or shock wave front, to subtend the wetting line it is necessary that

$$\frac{a \sin \alpha}{V'} \leq 1.$$

Thus the following cases are possible:

1) $\frac{V'}{\sin \alpha} < a$ impact is nonshock producing; or

2) $\frac{V'}{\sin \alpha} \geq a$ then impact shock occurs and either:

   a) $V' < a$ for which $\beta > \alpha$,

   b) $V' = a$ for which $\beta = \alpha$, or

   c) $V' > a$ for which $\beta < \alpha$

with the boundary requiring that $\beta \geq \alpha - \omega$, where $\beta = \alpha - \omega$ is equivalent to normal entry impact.

Critical Entry Angle

Equation (7) provides one necessary condition for water impact to be shock producing, namely, for a shock wave front to be established it is necessary that

$$\frac{V'}{\sin \alpha} > a, \tag{8}$$

or the speed of the disturbance across the surface of the water must be greater than the shock wave speed in water. The other necessary condition for water impact to be shock producing is that the wetting line speed across the face of the missile must be greater than the expansion wave speed across the projected rigid face, or

$$V = \frac{V'}{\tan \alpha} > c. \tag{9}$$

This is necessary for a nonzero shock pressure area, as can be seen in Figure 1.

The critical entry angle α^* is the entry angle of the missile beyond which the impact is no longer shock producing. From the two necessary conditions on shock impact, Equations (8) and (9), the critical entry angle will be the smaller of

$$\alpha^* = \min \begin{cases} \operatorname{Sin}^{-1}\left(\dfrac{V'}{a}\right) \\ \\ \operatorname{Tan}^{-1}\left(\dfrac{V'}{c}\right) . \end{cases} \tag{10}$$

<u>Component of Water Particle Velocity Normal to Shock Wave Front</u>

With the missile face deformation angle, resultant water particle velocity, and water shock wave front Mach angle found, the component of water particle velocity normal to the shock wave front, q, may now be determined. The geometry of these water particle velocity vectors is illustrated by Figure 6. With the resultant water particle velocity u perpendicular to the missile's deforming missile face and the normal component q perpendicular to the shock wave front the angle between them, γ, is the angle between the deforming missile face and the shock wave front, or

$$\gamma = \beta - (\alpha - \omega) . \tag{11}$$

Thus the normal component of water particle velocity is

$$q = u \cos \gamma . \tag{12}$$

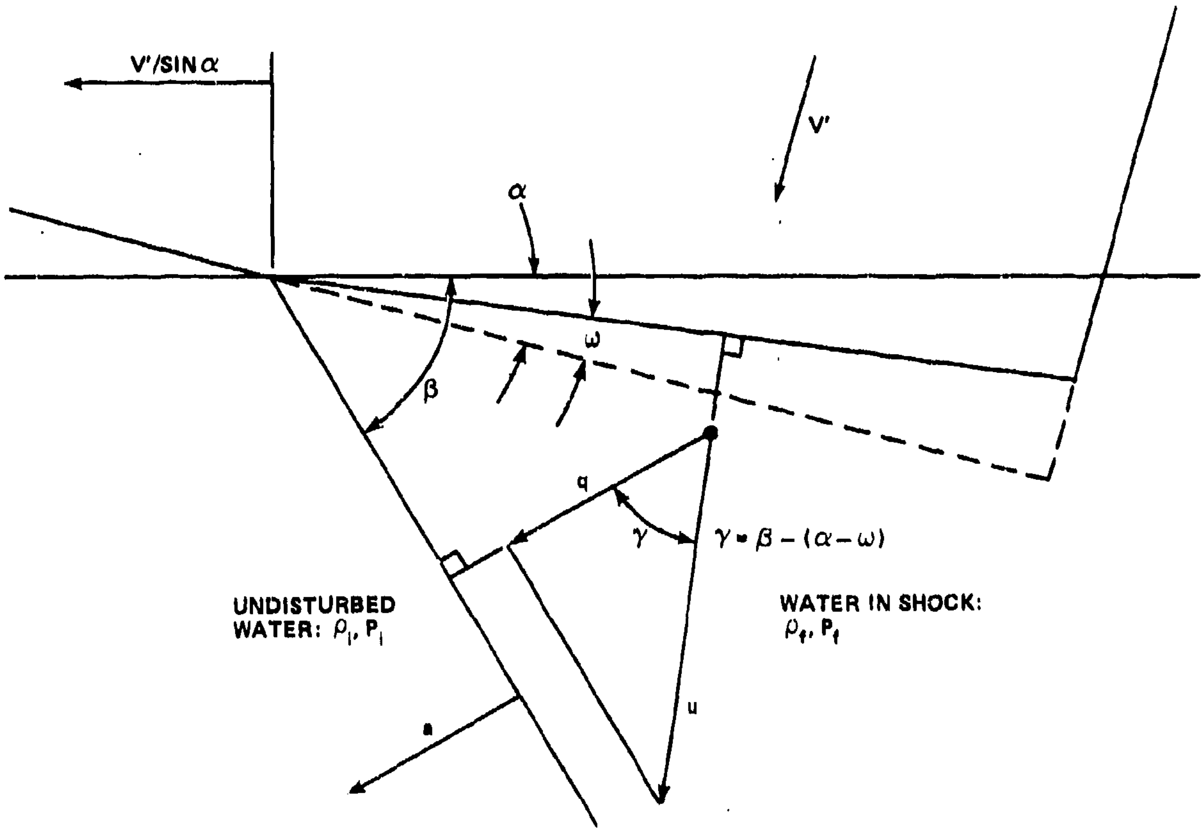

FIGURE 6. OBLIQUE SHOCK WAVE GEOMETRY

Note that for the special case of normal water entry impact, $\alpha=0$, Equations (5), (6), (7), (11), and (12) reduce to:

$$\omega = 0,$$

$$u = V' - \varepsilon_z a_m,$$

$$\beta = 0,$$

$$\gamma = 0, \text{ and}$$

$$q = u;$$

which is consistent with the results of D. Laumbach's paper.

RANKINE-HUGONIOT SHOCK PRESSURE

A disturbance of finite strength, a shock, will propagate through water as a compressive wave with a speed greater than the local speed of sound. These shocks move too fast for gross temperature changes to occur, i.e. adiabatically. Thus the relation between shock wave speed, pressure, and particle velocity may be derived by use of only the balance of momentum and the continuity equations. This is described by C.-S. Yih[6] and the derivation of the shock pressure, or Rankine-Hugoniot relation, follows below.

Derivation of the Rankine-Hugoniot Relation

Consider flow through the shock wave front shown in Figure 6. With the wave sweeping through a mass of water of unit cross sectional area in unit time we have from the -

Continuity Equation:

$$\rho_i a = \rho_f (a-q) \quad \text{and}$$

Momentum Balance:

$$P_f - P_i = \rho_i a^2 - \rho_f (a-q)^2 \ .$$

6 Yih, C.-S., Fluid Mechanics/A Concise Introduction to the Theory (New York: McGraw-Hill Book Co., 1969), pp. 266-273.

Substituting for $\rho_f(a-q)$ in the momentum equation from the continuity equation yields

$$p_f - p_i = \Delta p = \rho_i a^2 - \rho_i a(a-q)$$

$$\Delta p = \rho_i a\ [a - (a-q)]$$

$$\Delta p = \rho_i aq\ ,$$

which is the Rankine-Hugoniot relation. If p is taken to be the gauge shock pressure, and since $p_i \cong$ atmospheric, then $\Delta p = p$. Dropping the subscript on the initial or undisturbed mass density, the Rankine-Hugoniot relation becomes simply

$$p = \rho aq\ , \tag{13}$$

the impact shock pressure.

Determination of Shock Wave Speed in Water

Shock wave speed in water, a, increases with the associated shock pressure, with

$$a \rightarrow c \quad \text{as} \quad p \rightarrow 0.$$

Values of shock wave speed as a function of shock pressure have been determined by measurement on shocks in water. The paper by Rice and Walsh[7] tabulates the results of this experimental Hugoniot curve data for water initially at 20°C and 1 atmosphere to shock pressures of 250 kilobars. A plot of this data to 20 kilobars is given by Figure 7.

Computational use of this data table is facilitated by use of a four point Lagrangian interpolating polynomial, a description and formulation of which is given in C.F. Gerald's text.[8] If p is a shock pressure which lies within the table as: $p_1<p_2<p<p_3<p_4$, where p_1, p_2, p_3, and p_4 are successive shock pressure table entries with a_1, a_2, a_3, and a_4 as their corresponding shock wave speeds, then the interpolant is given by the cubic

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[7] Rice, M.H. and Walsh, J.M., "Equation of State of Water to 250 Kilobars," The Journal of Chemical Physics, Vol. 26, No. 4, 1957, p. 824.

[8] Gerald, C.F., Applied Numerical Analysis (Reading: Addison-Wesley Publishing Co., 1978), pp. 174-176.

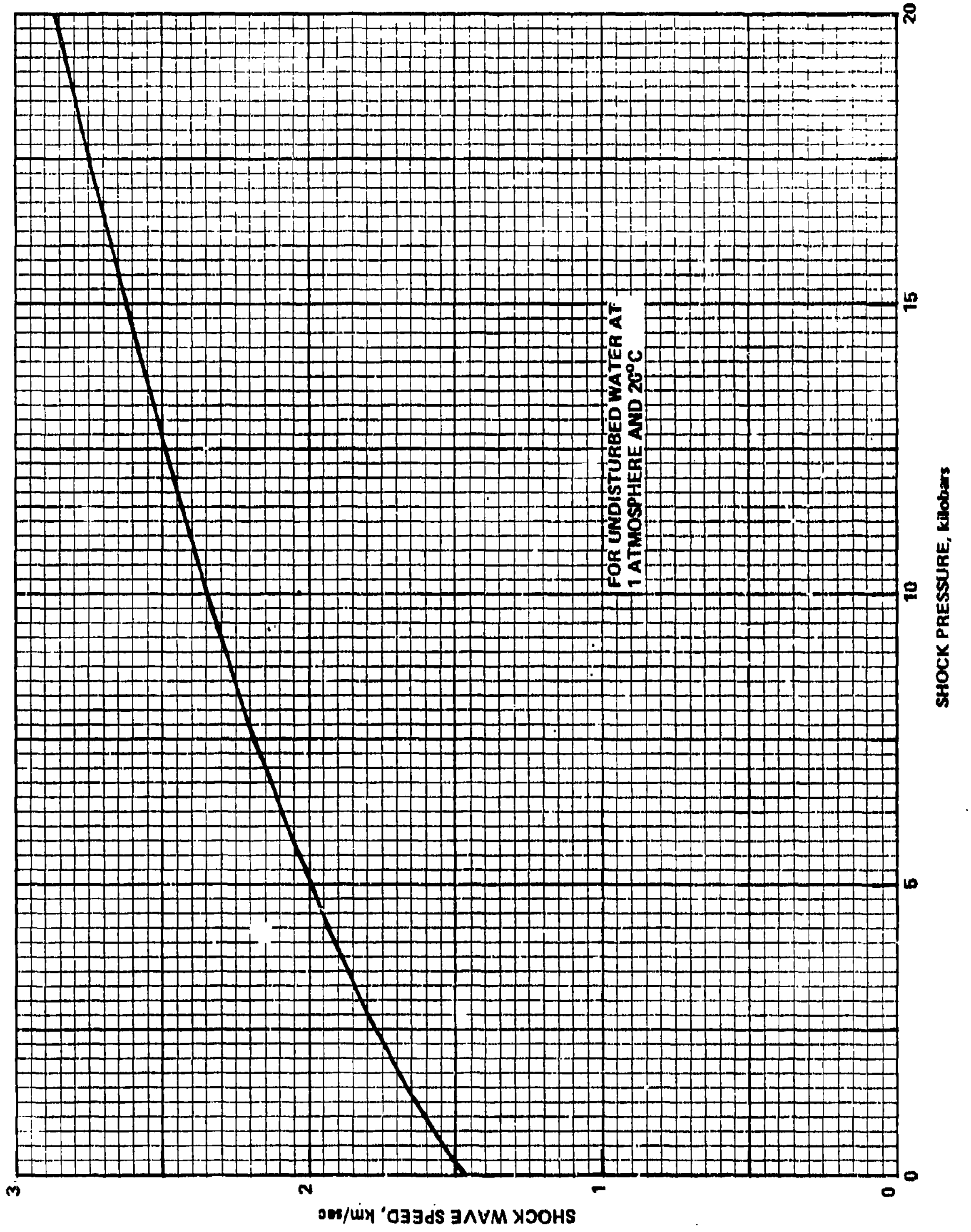


FIGURE 7. SHOCK WAVE SPEED IN WATER AS A FUNCTION OF SHOCK PRESSURE

$$a(p) = \frac{(p-p_2)(p-p_3)(p-p_4)}{(p_1-p_2)(p_1-p_3)(p_1-p_4)} a_1 + \frac{(p-p_1)(p-p_3)(p-p_4)}{(p_2-p_1)(p_2-p_3)(p_2-p_4)} a_2$$

$$+ \frac{(p-p_1)(p-p_2)(p-p_4)}{(p_3-p_1)(p_3-p_2)(p_3-p_4)} a_3 + \frac{(p-p_1)(p-p_2)(p-p_3)}{(p_4-p_1)(p_4-p_2)(p_4-p_3)} a_4. \tag{14}$$

Solution of the Rankine-Hugoniot Shock Relation

Since the shock wave speed is an empirical function of the shock pressure, and further, the water particle speed is a function of missile face deformation rate, and hence the shock pressure, the Rankine-Hugoniot relation,

$$p = \rho a(p)q(p) \;, \tag{R13}$$

is an implicit function of shock pressure.

Consider the following function of some trial pressure p_1 defined by

$$G(p_1) = p_1 - \rho a(p_1)q(p_1).$$

This is called the shock pressure error function since a solution of Equation (R13) for the shock pressure is a root of the error function, or

$$G(p) = p - \rho aq = 0. \tag{15}$$

A root is found by an iteration of trial pressures until Equation (15) is satisfied to within some specified tolerance. One iterative technique used for converging on a root of an implicit function is the Secant Method, which is described in a text by Atkinson,[9] with its first steps procedure illustrated by Figure 8.

The initial trial value of shock pressure used by the method is the weak or sonic shock pressure produced by a rigid body entering the water with speed V'. For this case the Rankine-Hugoniot relation reduces to the explicit form

$$p_1 = \rho cV'. \tag{16}$$

[9]Atkinson, K.E., An Introduction to Numerical Analysis (New York: John Wiley & Sons, 1978), pp. 48-52.

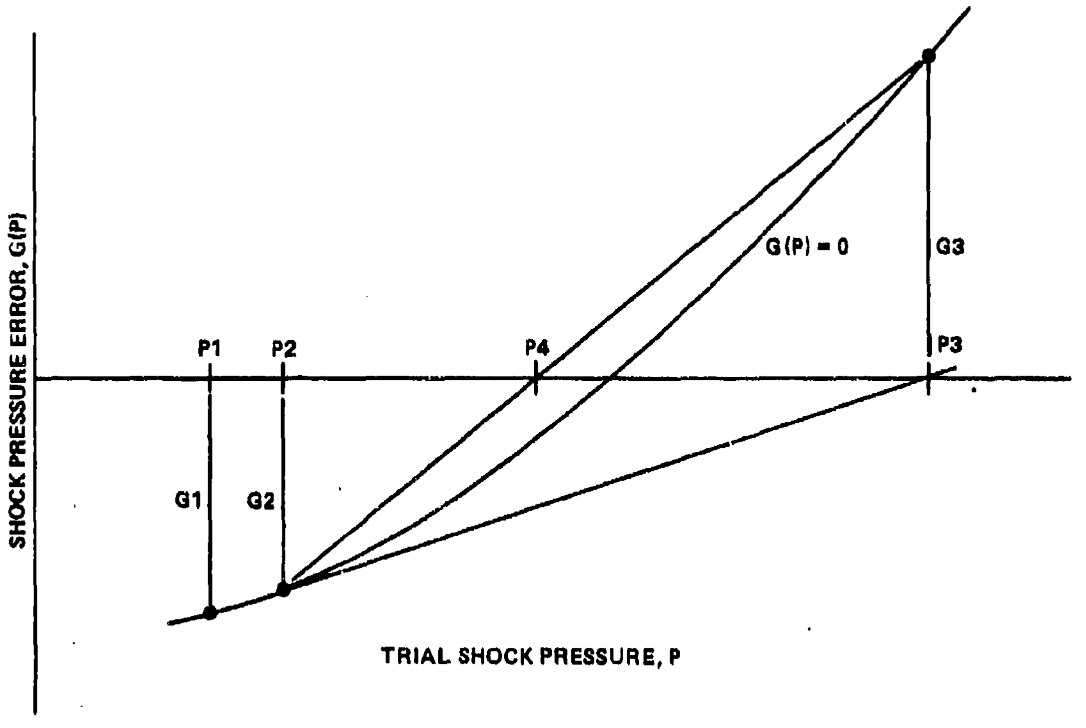


FIGURE 8. SECANT ITERATION TECHNIQUE

Since $c \leq a$ and $q \leq V'$ this trial pressure is a lower bound on the convergent shock pressure. A second trial value is needed to start the iteration process and this is supplied simply by an arbitrary 10% increase in the first trial value, i.e.,

$$p_2 = p_1 + 0.1\, p_1. \tag{17}$$

Between these two values on the $G(p)=0$ curve a secant is extended to intersect the curve at a point, determining the next trial pressure p_3. This process is then repeated using the latter two trial values to determine successive values according to the recursion formula

$$p_{i+1} = p_i - \frac{p_i - p_{i-1}}{G(p_i) - G(p_{i-1})} \quad G(p_i) \;,\; i = 3, 4, 5, \ldots, n \tag{18}$$

until convergence to within a specified accuracy on the ratio change in two successive values occurs, i.e. until

$$\left| \frac{p_{n+1} - p_n}{p_n} \right| \leq \text{error specified} \;. \tag{19}$$

EXPANSION WAVE SPEED ACROSS DEFORMING MISSILE FACE

During water entry the shock pressure behind the compression shock wave front is then later relieved back to the initial pressure with the passage of a rarefaction or expansion wave generated by the elastic recovery of the water. This expansion wave however travels only at the sonic or weak disturbance speed c' since water cannot sustain a tensile stress. Moving outward from the initial contact point this expansion wave moves across the deforming face of the missile with a component speed $b<c'$, since this face is not parallel to the local ray of the expansion wave. This component of expansion wave speed is determined by the trigonometry of Figure 9 as the face side bt opposite the expansion wave ray $c't$ and the extended deforming entry side $V't-\epsilon_z a_m t$.

The angle δ between the extended deforming face bt and the entry side $V't-\epsilon_z a_m t$ is the supplement of the complement of the deformation angle ω, as can be seen from the triangles in Figure 9, or

$$\delta = \pi - \left(\frac{\pi}{2} - \omega\right) \;,$$

$$\delta = \frac{\pi}{2} + \omega \;. \tag{20}$$

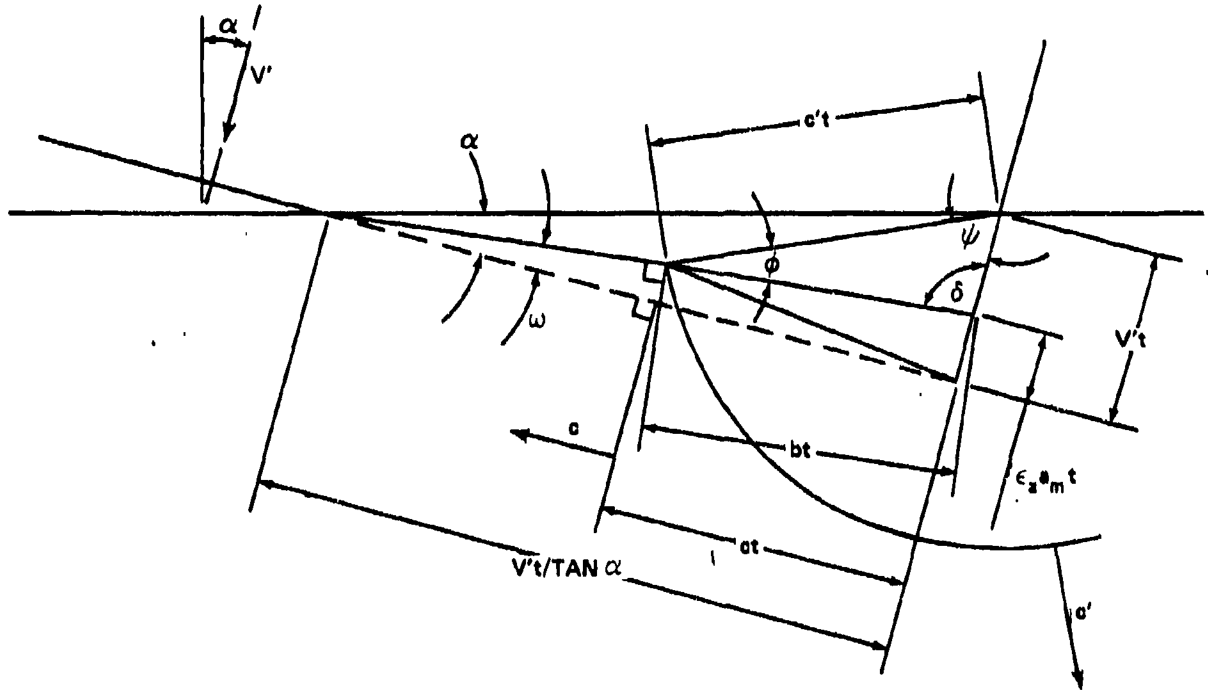


FIGURE 9. GEOMETRY OF EXPANSION WAVE MOVING ACROSS DEFORMING MISSILE FACE

The angle ϕ between the extended deforming face bt and the expansion wave ray c't is found by use of the Sine Law as

$$\phi = \sin^{-1}\left[\frac{(V' - \varepsilon_z a_m)\sin\delta}{c'}\right] \tag{21}$$

This leaves the angle ψ between the expansion wave ray and entry side as the triangle's supplement, or

$$\psi = \pi - (\delta + \phi) . \tag{22}$$

Finally employing the Sine Law again yields the deforming face component of expansion wave speed as

$$b = \frac{c' \sin\psi}{\sin\delta} .$$

The projection upon the rigid missile face of this speed is

$$c = b \cos\omega .$$

Thus the component of expansion wave speed moving across the undeformed circumferential circle of the missile's rigid face is

$$c = \frac{c' \sin\psi \cos\omega}{\sin\delta} . \tag{23}$$

This is the speed required for determining impact shock duration time, expansion wave profiles, and shock pressure areas relative to the missile's initial dimensions and geometric configuration, yielding results which will be projected axial components.

Note that for the special case of normal entry impact ($\alpha = 0$ implies $\omega = 0$ was previously shown) Equations (20), (21), (22), and (23) reduce to:

$$\delta = \frac{\pi}{2} ,$$

$$\phi = \mathrm{Sin}^{-1}\left(\frac{V' - \epsilon_z a_m}{c'}\right) ,$$

$$\psi = \frac{\pi}{2} - \mathrm{Sin}^{-1}\left(\frac{V' - \epsilon_z a_m}{c'}\right) , \text{ and}$$

$$c = c' \sin\left[\frac{\pi}{2} - \mathrm{Sin}^{-1}\left(\frac{V' - \epsilon_z a_m}{c'}\right)\right]$$

$$c = c'\left[\sin\frac{\pi}{2} \cdot \cos \mathrm{Sin}^{-1}\left(\frac{V' - \epsilon_z a_m}{c'}\right) - \cos\frac{\pi}{2} \cdot \sin \mathrm{Sin}^{-1}\left(\frac{V' - \epsilon_z a_m}{c'}\right)\right]$$

$$c = c' \cos \mathrm{Sin}^{-1}\left(\frac{V' - \epsilon_z a_m}{c'}\right)$$

$$c = c'\sqrt{1 - \left(\frac{V' - \epsilon_z a_m}{c'}\right)^2}$$

$$c = \sqrt{c'^2 - (V' - \epsilon_z a_m)^2} .$$

which is again consistent with the results of D. Laumbach's paper for normal entry impact.

TIME DURATION OF IMPACT SHOCK

When the expansion wave has engulfed the shock pressure region over the entire face of the missile the shock phenomena has ceased and only hydrodynamic forces are left. With the missile entering the water obliquely the wetting line moves across the rigid face at a finite speed V, e.g. see Figure, 1, where

$$V = \frac{V'}{\tan\alpha} \tag{24}$$

and with $V \to \infty$ as $\alpha \to 0$.

At this speed the wetting line will have traveled the missile's diameter in the time

$$T_e = \frac{2r}{V} = \frac{2r \tan \alpha}{V'} , \tag{25}$$

the water entry or full wetting time. At this point the initial expansion wave has traveled the distance cT_e leaving the remaining diametral length $2r - cT_e$ under shock pressure. This remaining length is then engulfed by both the initial and now the final expansion waves coming in from opposite diametral edges in the plane of symmetry as shown by Figure 10. This remaining length will be transversed by the two waves in the time $(2r - cT_e)/2c$. Thus the impact shock duration time will be the sum of this plus the water entry time, or

$$T_i = T_e + \frac{2r - cT_e}{2c} .$$

Substituting for T_e from Equation (25) and simplifying yields

$$T_i = \frac{r \tan \alpha}{V'} + \frac{r}{c} , \tag{26}$$

where $T_i\ (\alpha = 0) = r/c$ for normal impact, in which case expansion waves are generated on the full circumference simultaneously at $t = 0$ and there is no wetting line motion.

SHOCK AREA ON MISSILE FACE

In order to determine the impact shock force and impulse acting on the missile it is now necessary to determine the shock pressure area over the face of the missile. This area will be encompassed by the expansion wave profile on the face and the wetting line where the shock region had its origin, when this exists. This shock area profile is now determined for oblique and normal water entry. Note that these profiles will be constructed on the rigid face as a projection of the deformed face with the resulting shock areas, and hence forces, being axial components of the deformed face normal.

Shock Area Profile for Oblique Entry

For oblique entry a wetting line is established which moves across the undeformed face of the missile at finite speed V given by Equation (24). Chasing this wetting line is the expansion wave profile moving at speed c, given by Equation (23), with expansion wave rays being continuously generated on the circumference at the cusps of the wetting line. When the wetting line has

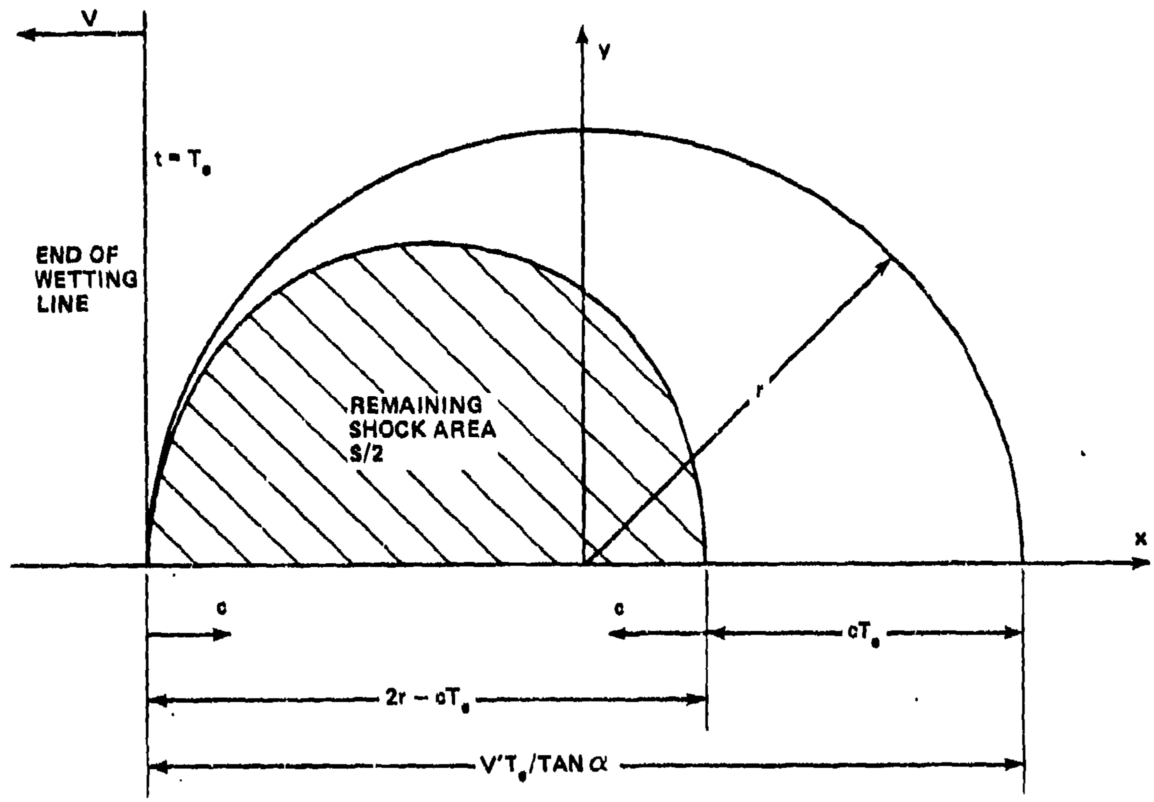


FIGURE 10. WATER ENTRY AND IMPACT SHOCK DURATION TIMES

traveled 2r to the end of the face, the face has become fully wetted and full water entry has occurred in the time T_e given by Equation (25). After this time no additional expansion wave rays are generated and the remaining shock area is engulfed by the then existing full circumferential expansion wave by impact shock duration time T_i, after initial water contact, given by Equation (26).

During Wetting Entry. For an instantaneous time since initial water contact of $t<T_e$ the shock area envelope is bounded by a wetting line, as shown in Figure 11, as well as the expansion wave profile. This profile is an envelope of the parametric family of expansion wave rays generated along the circumference between the initial contact point and the current cusps of the wetting line. From the initial contact point the ray which was generated at parametric time $\tau=0$, now has the instantaneous radius $\rho(\tau=0) = ct$. At the wetting line circumferential cusp an expansion wave ray is just being generated, thus its radius is $\rho(\tau=t) = 0$. So that at an intermediate parametric time τ, $0\leq\tau\leq t$, the wetting line occupied the position (ξ,η) on the circumference, having traveled the distance $V\tau$, and generated the expansion wave ray whose radius at current time t is now

$$\rho(\tau) = c(t-\tau) \tag{27}$$

with a missile circumferential origin in the (x,y) face coordinates of

$$\xi = r - V\tau \quad \text{and} \tag{28}$$

$$\eta = \sqrt{r^2 - \xi^2} \; . \tag{29}$$

These expansion wave rays generate the parametric family of circles:

$$(x - \xi)^2 + (y - \eta)^2 - \rho^2 = 0 \; .$$

The expansion wave profile on the rigid missile face will then be the curve tangent to this family of expanding circles. The necessary conditions which such an envelope E, of a family of curves C: $f(x,y,\tau) = 0$, must satisfy are developed in R. Courant's text.[10] The envelope must satisfy the system E:

1) $f(x,y,\tau) = 0$ and

2) $f_\tau(x,y,\tau) = 0$.

[10]Courant, R., Differential and Integral Calculus, Vol. II (New York: Interscience Publishers, 1936), pp. 171-179.

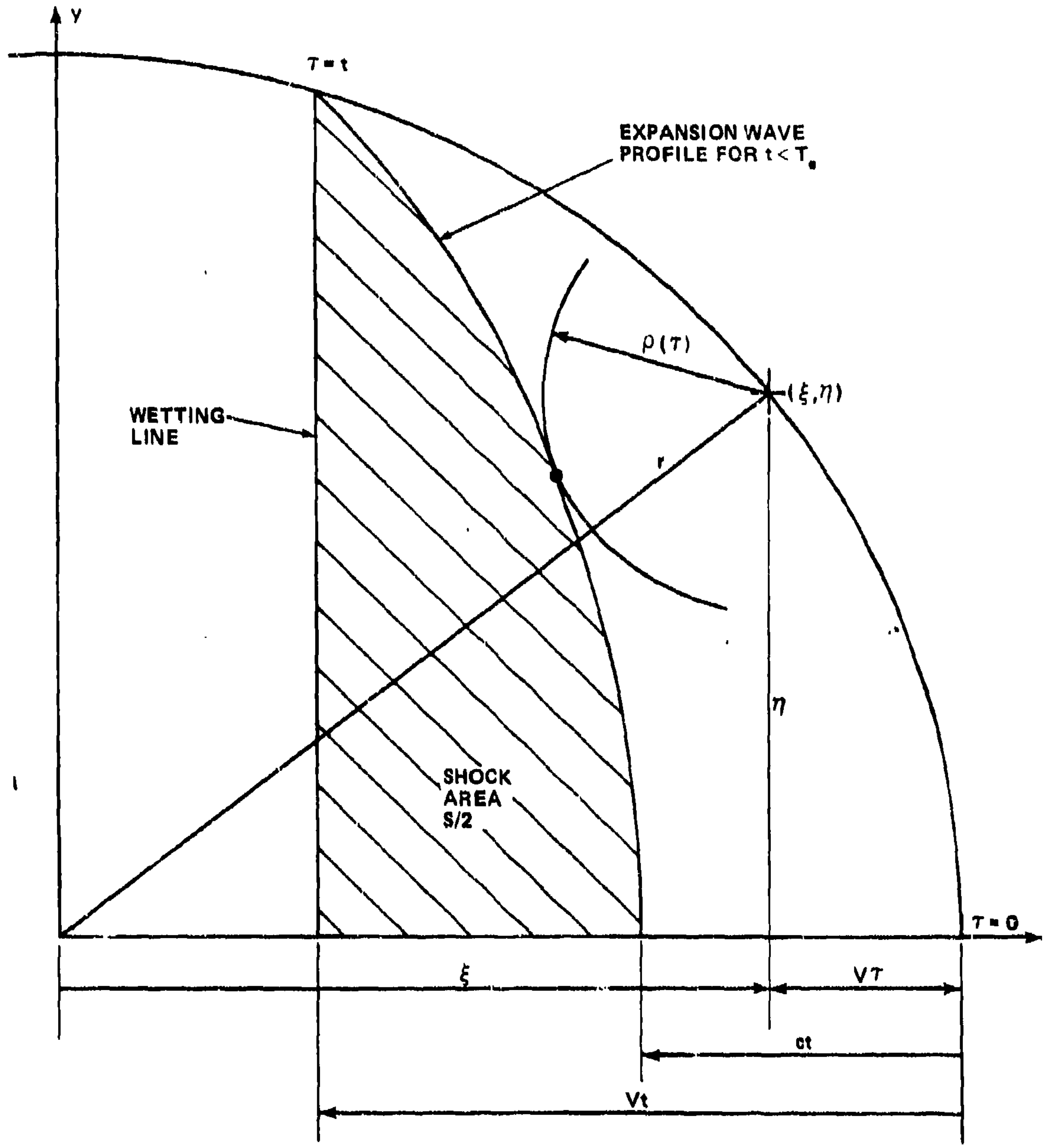


FIGURE 11. SHOCK AREA ENVELOPE DURING WETTING ENTRY

with

$$f(x,y,\tau) = (x - \xi)^2 + (y - \eta)^2 - \rho^2 = 0$$

then

$$\frac{\partial f}{\partial \tau} = 2(x - \xi)\left(- \frac{\partial \xi}{\partial \tau}\right) + 2(y - \eta)\left(- \frac{\partial \eta}{\partial \tau}\right) - 2\rho \frac{\partial \rho}{\partial \tau} = 0,$$

where

$$\frac{\partial \xi}{\partial \tau} = -V \ ,$$

$$\frac{\partial \eta}{\partial \tau} = \frac{1}{2} (r^2 - \xi^2)^{-\frac{1}{2}} \cdot (-2)\xi \frac{\partial \xi}{\partial \tau} \quad \text{or}$$

$$\frac{\partial \eta}{\partial \tau} = \frac{\xi}{\eta} V, \text{ and}$$

$$\frac{\partial \rho}{\partial \tau} = -c.$$

Thus

$$\frac{\partial f}{\partial \tau} = (x - \xi) + (y - \eta)\left(- \frac{\xi}{\eta}\right) - \frac{\rho(-c)}{V} = 0$$

or, with $\delta = \frac{c}{V} \rho$, (30)

the necessary conditions for the envelope E become:

1) $(x-\xi)^2 + (y-\eta)^2 - \rho^2 = 0$ and

2) $x - \frac{\xi}{\eta} y + \delta = 0$.

Solving condition (2) for y gives

$$y(\tau) = \frac{\eta}{\xi}\,(x + \delta)\ . \tag{31}$$

Then substituting this expresion for y into condition (1), collecting like terms, and making use of the relation $\xi^2 + \eta^2 = r^2$ yields the quadratic in x:

$$r^2x^2 - 2(r^2\xi - \eta^2\delta)x + [r^2\xi^2 - \eta^2\delta\ (2\xi - \delta) - \xi^2\delta^2] = 0.$$

With

$$A = r^2,$$

$$B = -2(r^2\xi - \eta^2\delta), \text{ and}$$

$$C = r^2\xi^2 - \eta^2\delta\ (2\xi - \delta) - \xi^2\rho^2$$

the roots will be

$$x_{1,2} = -\left(\frac{B}{2A}\right) \pm \sqrt{\left(\frac{B}{2A}\right)^2 - \frac{C}{A}}\ ,$$

where

$$-\left(\frac{B}{2A}\right) = \frac{r^2\xi - \eta^2\delta}{r^2} \quad \text{and}$$

$$\left(\frac{B}{2A}\right)^2 - \frac{C}{A} = \frac{\xi^2(r^2\rho^2 - \eta^2\delta^2)}{r^4} \geq 0.$$

This discriminate will be positive, and hence $x(\tau)$ real, if $\rho \geq \delta$ (where $r \geq \eta$ by definition) or, from Equations (27) and (30), if $V \geq c$. This is the wetting line speed condition required for a nonzero shock pressure area to be established, cf. Equation (9).

Of the two real roots, x_1 and x_2, the smaller is the one of interest since it lies within the missile's circumference. Thus

$$x(\tau) = \xi - \frac{\eta^2\delta + \xi\sqrt{r^2\rho^2 - \eta^2\delta^2}}{r^2}. \tag{32}$$

Equations (32) and (31) are the parametric coordinates of the expansion wave profile projected over the missile's rigid face, $[x(\tau), y(\tau)]$ for $0 \leq \tau \leq t$, at a current instantaneous impact time t since initial water contact.

After Face is Fully Wetted. For $t > T_e$ the missile's face is fully wetted, the wetting line no longer exists, and established expansion waves emanate from the full circumference of the face as shown by Figure 12. Since no new expansion wave rays are generated the established ones will have their circumferential origins as previously described by Equations (28) and (29) when $t = T_e$, or

$$[\xi(\tau), \eta(\tau)] \quad , \quad 0 \leq \tau \leq T_e \text{ for } t \geq T_e .$$

With the waves continuing to expand from their established positions with speed c, Equation (27) for their parametric radius remains unchanged as

$$\rho(\tau) = c(t - \tau) \; , \quad 0 \leq \tau \leq T_e \text{ for } t \geq T_e .$$

Hence the expansion wave profile $[x(\tau), y(\tau)]$ is described by the same set of equations, Equations (32) and (31), as for the wetting entry but with the time parameter limited accordingly as

$$0 \leq \tau \leq \begin{cases} t & \text{for } t < T_e \\ T_e & \text{for } t \geq T_e . \end{cases} \tag{33}$$

Integration for the Shock Pressure Area

With the envelope of the shock pressure area on the missile's rigid face-- the expansion wave profile described, this region may now be integrated to determine the projected shock pressure area on the rigid face. This integration is performed numerically with equal time steps in the time parameter τ describing the profile. These time steps are a sufficiently large division of the current impact time t, or the water entry time T_e if $t \geq T_e$, so as to insure sufficient numerical accuracy. Since equal steps in the parameter of a parametric curve do not, in general, result in equal divisions of the x-ordinate, the standard numerical integration schemes based upon equal interval divisions of the ordinate cannot be used. Integration is instead based on a generalization of Simpson's Rule for parametric curves, illustrated by Figure 13, in which a

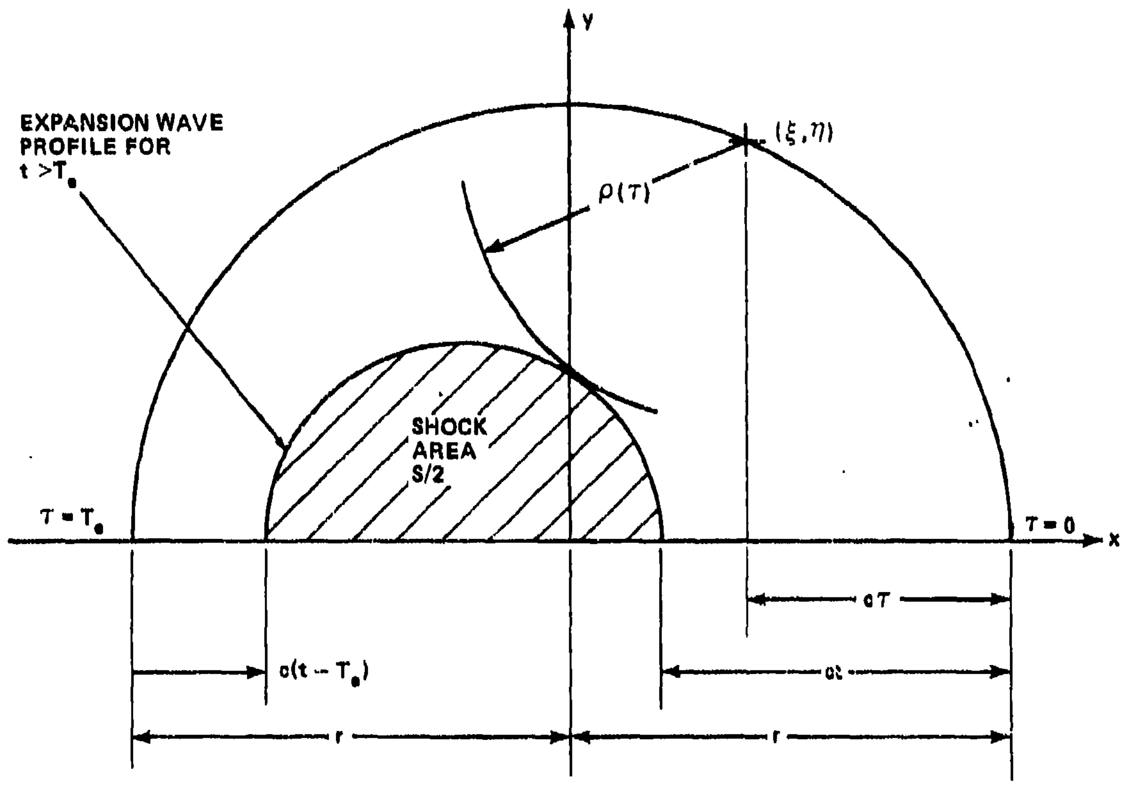
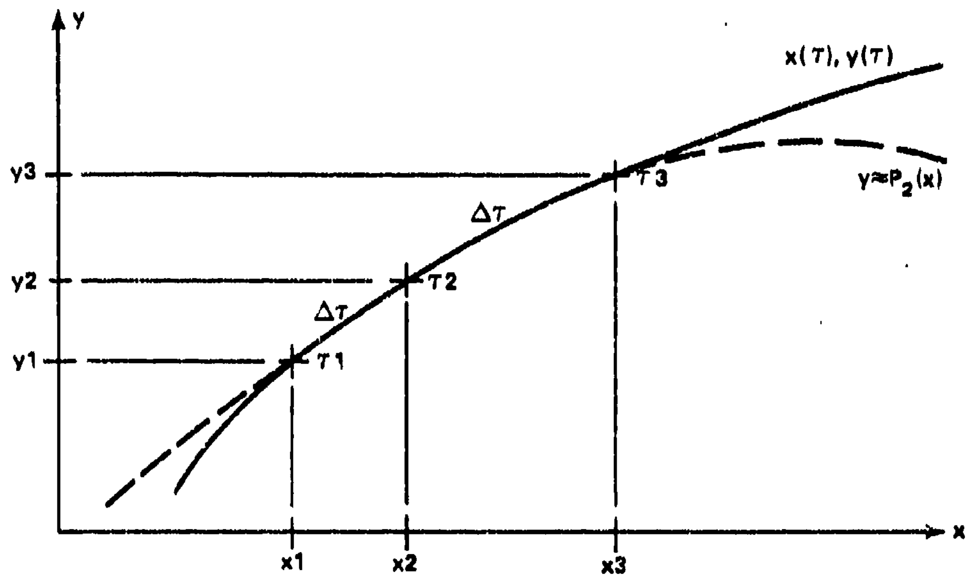


FIGURE 12. SHOCK AREA ENVELOPE AFTER FULL WATER ENTRY



EQUAL PARAMETER STEPS $\Delta\tau$ RESULT IN
UNEQUAL ORDINATE DIVISIONS Δx

FIGURE 13. SIMPSON'S RULE INTEGRATION FOR PARAMETRIC CURVE

Lagrangian second order interpolating polynomial $P_2(x)$ is passed through three successive points on the parametric curve, approximating that curve and corresponding to a single integration panel.

If (x_1,y_1), (x_2,y_2), and (x_3,y_3) are three successive points on the expansion wave profile curve corresponding to the successive time parameters τ_1, $\tau_2 = \tau_1 + \Delta\tau$, and $\tau_3 = \tau_1 + 2\Delta\tau$ respectively, then the Lagrangian interpolating polynomial for these three points is

$$P_2(x) = \frac{(x-x_2)\ (x-x_3)}{(x_1-x_2)\ (x_1-x_3)}\, y_1 + \frac{(x-x_1)\ (x-x_3)}{(x_2-x_1)\ (x_2-x_3)}\, y_2 + \frac{(x-x_1)\ (x-x_2)}{(x_3-x_1)\ (x_3-x_2)}\, y_3 .$$

Integrating this for the upper half profile curve, with $x(\tau)$ and $y(\tau)$ provided by Equations (32) and (31) respectively, for $0 \le \tau \le t$, or T_e if $t \ge T_e$, yields and approximation of half the shock pressure area, because of symmetry about the x-axis, for a single panel, or

$$\frac{\Delta S_1}{2} = \int_{x_1}^{x_3} P_2(x)dx.$$

which, after carrying out the integration, results in the area approximating formula

$$\frac{\Delta S_1}{2} = \frac{\frac{1}{3}(x_3^3 - x_1^3) - \frac{1}{2}(x_2 + x_3)(x_3^2 - x_1^2) + x_2x_3(x_3 - x_1)}{(x_1 - x_2)(x_1 - x_3)}\, y_1$$

$$+ \frac{\frac{1}{3}(x_3^3 - x_1^3) - \frac{1}{2}(x_1 + x_3)(x_3^2 - x_1^2) + x_1x_3(x_3 - x_1)}{(x_2 - x_1)(x_2 - x_3)}\, y_2$$

$$+ \frac{\frac{1}{3}(x_3^3 - x_1^3) - \frac{1}{2}(x_1 + x_2)(x_3^2 - x_1^2) + x_1x_2(x_3 - x_1)}{(x_3 - x_1)(x_3 - x_2)}\, y_3 . \qquad (34)$$

Since two increments of the time parameter are required per panel the number of division steps must be even. Thus for a given instantaneous impact time t

$$\frac{S}{2} = \sum_{i=1,3,5,\ldots}^{n} \frac{\Delta S_i}{2}, \quad 0 \le t \le t_i ,$$

where $i = 1$ corresponds to $\tau = 0$ and

$$i = n \text{ corresponds to } \tau = \begin{cases} t \text{ for } t < T_e \\ T_e \text{ for } t \ge T_e \end{cases}$$

with $\Delta\tau = t/n$ or T_e/n and n is odd, i.e. there are n-1 even time parameter intervals.

The Special Case of Normal Entry

For the case of normal water entry impact, $\alpha = 0$, expansion waves are generated on the full circumference of the missile's face instantly, at $t = 0$, and simultaneously. This results in a concentric circular expansion wave profile shrinking the shock pressure area with speed c. Thus the shock pressure area for this case is simply the area of a circle with a radius diminishing at the rate c, or

$$S(\alpha = 0) = \pi \, (r - ct)^2 , \quad 0 \le t \le T_i \tag{35}$$

IMPACT SHOCK FORCE AND IMPULSE

Finally with the impact shock pressure and instantaneous projected shock pressure area determined, the axial component of the impact shock force acting on the missile is

$$F(t) = pS(t). \tag{36}$$

With this result the total axial shock impulse may also be found as

$$I = \int_0^{T_i} F(t)dt.$$

This integral is evaluated numerically on equal intervals of t by use of Simpson's 1/3 Rule:

$$I = \frac{\Delta t}{3} \sum_{i=1,3,5,\ldots}^{n} (F_i + 4F_{i+1} + F_{i+2}) , \tag{37}$$

where $i = 1$ corresponds to $t = 0$,

$i = n$ corresponds to $t = T_i$,

and $\Delta t = T_i/n$ for n odd.

As before n-1 even time intervals are required for the approximate integration, with a larger value of n improving the accuracy on the total impulse.

IMPACT SHOCK DYNAMICS COMPUTATION PROGRAM 'IMPACT'

Computation of the shock dynamics for a flat faced cylindrical missile impacting the water's surface is carried out using the numbered equations developed in the previous section by the program IMPACT. A listing of this program is given in Appendix A. The programming language used is CDC's version of FORTRAN IV. The program is then applied to a hypothetical, but hopefully typical, example problem of a missile entering the water at various angles. Output for this example is given by Appendix B. The results of this example are then discussed with regard to water impact physics and the influence of entry angle.

DESCRIPTION OF PROGRAM STRUCTURE

The program IMPACT has been written in a simplistic or "engineering" format with emphasis upon clarity and ease of modification rather than efficiency. The program structure is linear, following the same sequence of steps used in the derivation of the basic equations for impact shock dynamics. An outline of the program code is given below, with the program symbolic variable, its corresponding equation symbol, and its equation number in the previous sections included.

I. CONSTANT PHYSICAL DATA ENTRY (Automatic)
   - A. Rice & Walsh Shock Wave Speed Data
     1. Shock pressure, D(1,I) = p, psi
     2. Shock wave speed, D(2,I) = a, in/sec.
   - B. Undisturbed Properties of Water
     1. Geometric pi, PI = π
     2. Density, RHO = ρ, lb-sec^2/in^4
     3. Speed of sound, C1 = c', in/sec.

II. ENTRY OF INDEPENDENT VARIABLES (Prompted)
   A. Missile Radius, R = r, in
   B. Impact Velocity, V2 = V', fps
   C. Entry Angle, ALPHA1 = α, deg.
   D. Missile Weight Density, RHOM1 = ρ_m/g, lb/in^3
   E. Young's Elastic Modulus, E = E, psi
   F. Poission's Ratio, NU = ν
   G. Number of Time Intervals for Printout, M = n/2

III. PRINTING OF TABLE HEADER
   A. Title
   B. Input Data Review

IV. COMPUTATION OF CONSTANT SHOCK DYNAMICS RESULTS
   A. Missile Elastic Wave Speed, AM = a_m (3)
   B. Shock Pressure Computation
      1. Initial Trial Shock Pressure, P1 = p_1 (16)
      2. Subroutine PRESS:
         a. Axial missile strain, EZ = ϵ_z (4)
         b. Missile face deformation Angle OMEGA = ω (5)
         c. Resultant water particle speed, U = u (6)
         d. Subroutine ITABLE: Interpolation for water shock wave speed data from trial shock pressure (14)
         e. Check on surface disturbance speed for shock (8)
         f. Mach angle, BETA = β (7)
         g. Normal water particle speed, Q = q (12)
         h. Rankine-Hugoniot shock pressure, PS = p (13)
         i. Shock pressure error function, G = G(p) (15)
      3. Second Trial Shock Pressure, P2 = p_2 (17)
   C. Subroutine SECANT: New iterate value for shock pressure (18)
   D. Expansion Wave Speed Across Rigid Face, c
      1. DELTA = δ (20)
      2. PHI = ϕ (21)
      3. PSI = ψ (22)
      4. C = c (23)
   E. Time at Full Wetting Entry, TE = T_e (25)
   F. Time Duration of Impact Shock, TI = T_i (26)

V. PRINTOUT OF CONSTANT SHOCK DYNAMICS RESULTS
   A. Shock Wave Speed in Missile, AM = a_m, in/sec.
   B. Elastic Strain on Missile Face, EZ = ϵ_z, in/in
   C. Deformation Angle of Missile Face, OMEGA1 = ω, deg.
   D. Water Particle Velocity in Shock, U = u, in/sec.
   E. Shock Wave Speed in Water, A = a, in/sec.
   F. Mach Angle of Shock Wave in Water, BETA1 = β, deg.
   G. Water Particle Speed Normal to Shock Wave, Q = q, in/sec.
   H. Rankine-Hugoniot Shock Pressure, P = p, psi
   I. Expansion Wave Speed Across Missile Face, C = c, in/sec.
   J. Time at Full Wetting Entry, TE = t_e, sec.
   K. Time Duration of Impact Shock, TI = T_i, sec.

VI. COMPUTATION OF TIME DEPENDENT RESULTS

A. Time Step Interval, H = Δt

B. Initialize Impulse, FT = I = 0.

C. Shock Pressure Area Determination, S(t)

1. For Normal Impact, S = S (35)
2. For Oblique Impact:
   a. Wetting line speed, V = V (24)
   b. Check on wetting line speed for shock (9)
   c. Subroutine AREA: Half area between wetting line and expansion wave profile
      i. Initialize area, A = S/2 = 0.
      ii. Determine time parameter limit, T1 (33)
      iii. Time parameter for profile curve, TAU = τ
      iv. XI = ξ (28)
      v. ETA = η (29)
      vi. RHO = ρ (27)
      vii. DELTA = δ (30)
      viii. Coordinates of expansion wave profile curve:
      X = $x(\tau)$ (32)
      Y = $y(\tau)$ (31)
      ix. Simpson's Rule integration for unequal ordinate intervals, A = S/2 (34)

D. Impact Shock Force, F = F(t) (36)

E. Subroutine SIMP: Integration of impact shock force over time for impulse, FT = I(t) (37)

VII. PRINTOUT OF SHOCK DYNAMICS RESULTS

A. Water Entry Time, T = t, sec.

B. Shock Pressure Area, S = S, in^2

C. Impact Shock Force, F = F, lb

D. Impact Shock Impulse, FT = I, lb-sec.

The error tolerance on convergence to a root, used with the subroutine SECANT, as a ratio change in its previous value is 0.0001. For the numerical integration of shock pressure area, carried out in the subroutine AREA, 200 subdivisions of the parametric time interval is used. This has been found by trial to result in a lower bound on area with an error of about 5 parts in 100,000. Finally, the number of steps used for integration of the total shock impulse is twice the number of time intervals of shock duration time requested of the user for the printing of the time dependent results. With this integration carried out by Simpson's Rule in the subroutine SIMP, the error will be, from Gerald, C.F., "Applied Numerical Analysis", p. 215,

$$\text{Error} = -\frac{T_i}{180}\left(\frac{T_i}{2M}\right)^4 F^{iv}(t) \;,\; 0 \le t \le T_i \;,$$

where: T_i is the impact duration time,

M the number of time intervals for printout requested by the user, and

$F^{iv}(t)$ is the fourth time derivative of the impact shock force (which unfortunately is not known prior to computation).

The program makes checks on the conditions necessary for the impact to be shock producing in accordance with Equations (8) and (9). If either of these conditions is not satisfied then the appropriate message is printed and execution halts. Thus the progam IMPACT will give shock dynamics results only for those entry angles which are less than the critical entry angle.

USE OF THE PROGRAM

The program has been written for interactive use on a time sharing terminal with the program normally kept in the users library as an active file, or loaded into it by a card deck. All the necessary independent variables needed to describe a particular water entry impact problem are prompted for by the running program in typical engineering units, specified in the prompt as:

- ENTER RADIUS OF MISSILE, inches
- IMPACT VELOCITY OF MISSILE, feet per second
- ANGLE OF ENTRY FROM VERTICAL, degrees
- ENTER WEIGHT DENSITY OF MISSILE, pounds per cubic inch
- YOUNG'S ELASTIC MODULUS, pounds per square inch
- POISSION'S RATIO FOR MISSILE, (dimensionless)

In addition to the independent variables the number of time intervals in the impact shock duration time for print out is requested of the user as:

- ENTER THE NUMBER OF TIME INTERVALS DESIRED =

Since the integration of shock force over time is accomplished by the numerical method of Simpson's Rule, a larger number of intervals entered here yields a better computational accuracy on the total shock impulse. With the integration steps used internally being twice the number of printing steps prompted, an entry of about 50 time intervals is generally sufficient for engineering accuracy. An increase to 100 resulted in only a 0.01% change in total impulse for the example with $\alpha = 5^o$.

With the number of time intervals for printout entered the program proceeds without further action by the user. The input data is now tabulated and the constant shock dynamics parameters are computed, including the Rankine-Hugoniot shock pressure. During this time a check is made on the surface disturbance speed as a condition for an impact shock to exist. This must be greater than the shock wave speed in accordance with Equation (8). If this is not satisfied a message is printed indicating so and program execution halts. Otherwise the program proceeds with a printing of the constant shock dynamics results and begins computation for the time dependent results. Prior to determination of the shock pressure area a check is also made on the wetting line speed. This must be greater than the expansion wave profile speed for a shock pressure area to exist as specified by Equation (9). If this condition is not met a message is printed and the program stops. Otherwise execution proceeds with computation and printing of the time dependent results for each increment of the impact shock duration time: shock pressure area, force, and impulse; with the total shock impulse being the last value of impulse printed.

A sample terminal session in the use of the program IMPACT is given by Appendix B for the example problem with a 5° entry angle. Included are the prompts for input, the input data used, and the resulting output tables of constant and time dependent impact shock dynamics.

EXAMPLE PROBLEM

For the purposes of illustrating the use of the program and also for verification, by the physical response, the assumptions used in developing the basic equations, a hypothetical impact problem is solved with IMPACT. This example problem is the water entry impact of a flat faced cylindrical missile of aluminum with a 6 inch radius entering the water at a speed of 1,500 feet per second. The program IMPACT will then be run for various entry angles so that shock force and impulse as functions of entry angle as well as of time can be found. The necessary input data required for this problem will then be:

Missile Radius = 6.0 in,
Impact Velocity = 1500 fps,
Weight Density of Missile = 0.1 lb/in^3,
Young's Modulus = 10E6 psi, and
Poission's Ratio = 0.333;

with the entry angle varied for each run from 0 to 12 degrees.

RESULTS OF THE EXAMPLE

The impact shock force as a function of water entry time has been plotted for entry angles of 0, 1, 2, 5, and 10 degrees in Figure 14. From this graph it is seen that maximum shock force occurs for normal entry (0 deg. entry angle) at the instant of water contact and diminishes, with an increasing time delay, with increasing entry angles. So that as the entry angle approaches zero the impact shock force-time profile approaches the normal entry case. The maximum impact shock force is plotted for entry angle in Figure 15. This again illustrates the uniform decrease in shock force with an increase in entry angle from a maximum for normal impact to, what would be, zero for the critical entry angle, greater than the 12 degrees shown. Figure 16 graphs the total impact shock impulse as a function of entry angle. This shows the total impulse diminishing with entry angle as expected, though gradually at first because of the corresponding increase in shock duration time.

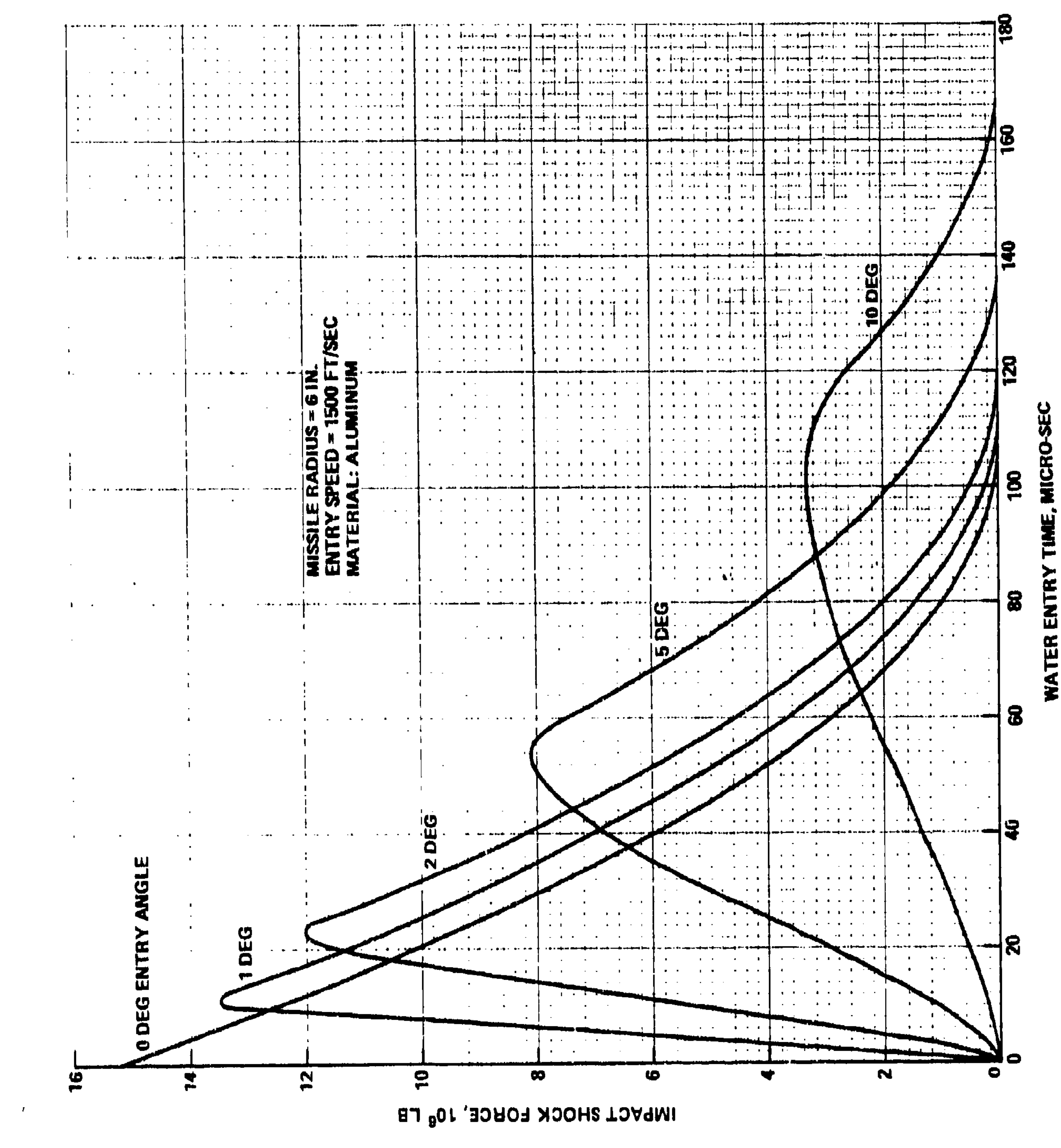


FIGURE 14. IMPACT SHOCK FORCE AS A FUNCTION OF WATER ENTRY TIME

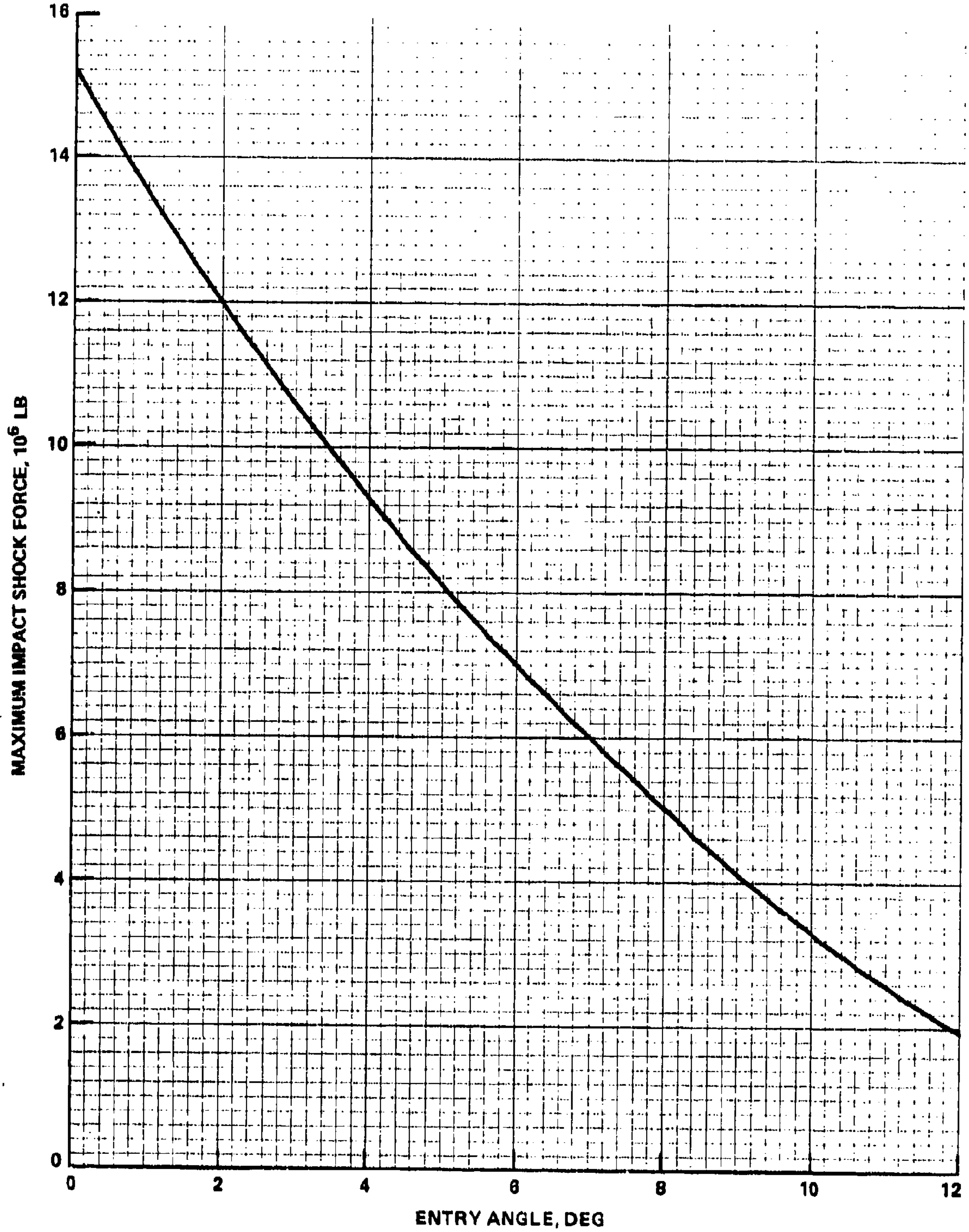


FIGURE 15. MAXIMUM IMPACT SHOCK FORCE AS A FUNCTION OF ENTRY ANGLE

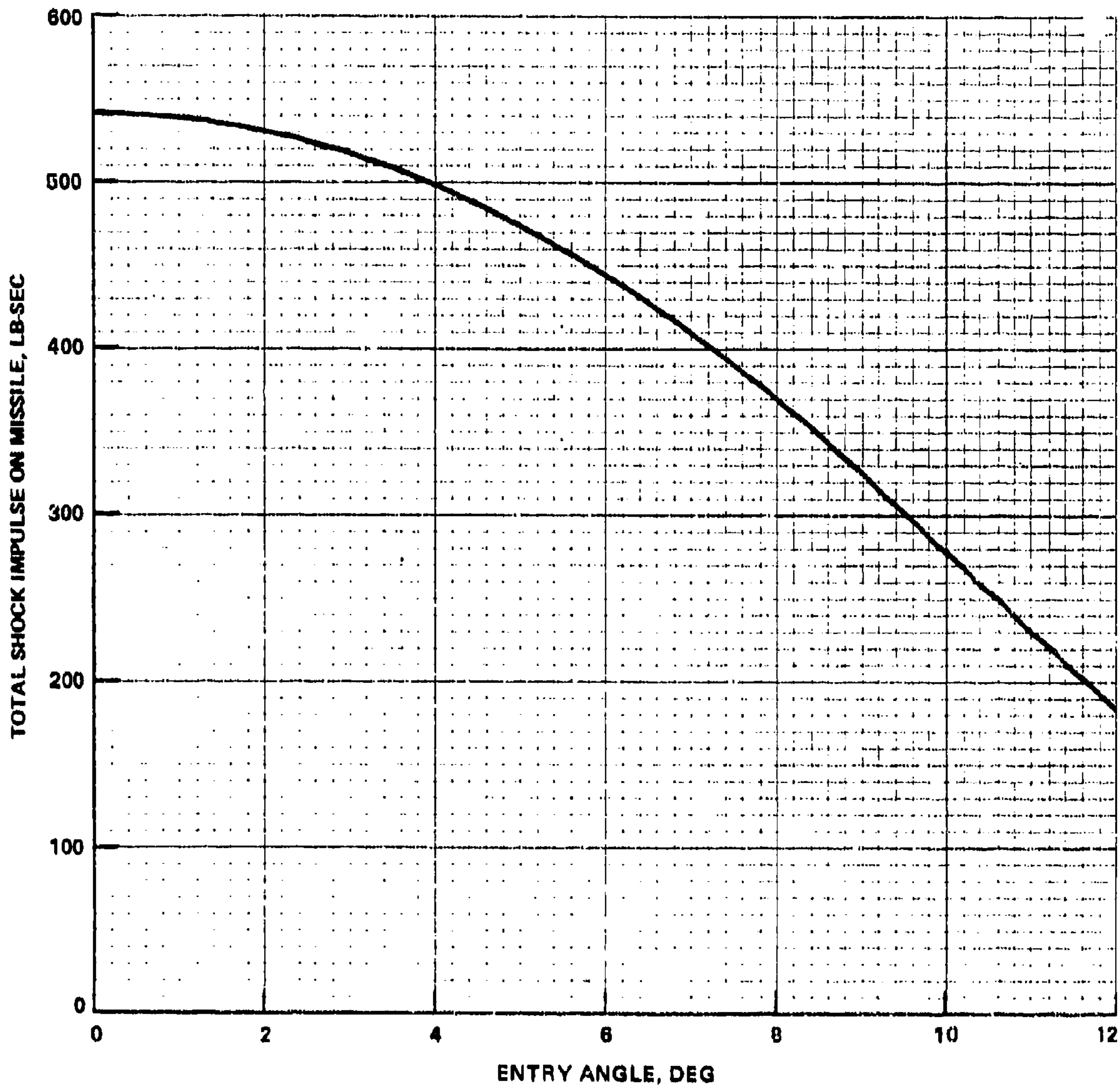


FIGURE 16. TOTAL IMPULSE AS A FUNCTION OF ENTRY ANGLE

These results illustrate a physical response one would expect of water entry shock producing impacts. The results also converge to those for normal entry impact as given by D.D. Laumbach's original paper. However, in order to determine whether the program provides results which are adequately approximate it is necessary to compare them with experimental tests of oblique water impact shocks on model missiles.

POSSIBLE MODIFICATIONS AND IMPROVEMENTS TO THE PRESENT FORMULATION

The most important continuation of this study is an experimental verification, for oblique entry shock, of the formulated model and program IMPACT. Beyond this the following improvements are possible extensions which can be carried out in the simplified approach used thus far:

1) Let the entry velocity be variable. Its change is proportional to the instantaneous impulse being computed and inversely to the missile's mass.

2) Let the entry angle be variable during impact shock. Its acceleration is proportional to the torque produced by the shock force. This requires finding the centroid of the shock pressure area and adding a moment of inertia for the missile to the input data.

3) The assumptions of pure axial deformation and constant face deformation speed under shock pressure requires further consideration with regard to the theory of elasticity and wave mechanics.

4) An analytical expression for the empirical shock wave speed-shock pressure data for water would be a convenient substitute for the interpolated table.

5) A program for determining the critical entry angle for a given missile entering the water at a given speed would be useful in establishing limits on the entry angles for shock producing impacts.

NOMENCLATURE

a	Shock wave speed in water (in/sec.)
b	Expansion wave speed across deformed missile face (in/sec.)
c	Expansion wave speed across deformed missile face relative, or projected, to the rigid face (in/sec.)
c'	Expansion wave speed in water, speed of sound (in/sec.)
E	Young's elastic modulus of missile material (psi)
$F(t)$	Impact shock force on missile face (lb)
$G(p)$	Shock pressure error function (psi)
$I(t)$	Impact shock impulse on missile (lb-sec.)
n	Number of time steps of impact shock duration time used for integration of impulse
p	Impact shock gauge pressure (psi)
p_i	Initial or undisturbed water pressure (psi)
p_f	Final or shocked water pressure (psi)
q	Component of shocked water particle velocity normal to water shock wave front (in/sec.)
r	Radius of cylindrical missile (in)
S	Shock pressure area on missile face (in^2)
t	Time since missile first touches the water (sec.)
T_e	Time at full wetting of missile's face (sec.)
T_i	Time duration of impact shock (sec.)
u	Resultant shocked water particle velocity relative to undisturbed water (in/sec.)

NOMENCLATURE (Cont.)

V	Wetting line speed across missile's face in rigid face plane (in/sec.)
V'	Missile's axial water entry velocity (in/sec.)
w	Axial displacement of deformed face projected to initial entry side (in)
$\dot{w}$	Axial speed of deforming face (in/sec.)
$x(\tau)$	Ordinate of expansion wave profile curve on rigid missile face (in)
$y(\tau)$	Abscissa of expansion wave profile curve on rigid missile face (in)
α	Entry angle of missile's axis from a normal to the water's surface (rad.)
α^*	Critical entry angle (rad.)
β	Mach angle of water shock wave front (rad.)
γ	Angle between water shock wave normal and water particle velocity vector (rad.)
δ	Angle between deformed missile face and side of missile in plane of symmetry (rad.)
$\delta(\tau)$	Expansion wave profile curve translation parameter (in)
Δp	Change in water pressure across a shock wave front (lb/in^2)
$\Delta\rho$	Change in water density across a shock wave front ($lb\text{-}sec^2/in^4$)
ϵ_x, ϵ_y	Lateral and transverse strains in shocked miscile (in/in)
ϵ_z	Axial strain in missile due to impact shock (in/in)
$\eta(\tau)$	Ordinate of an expansion wave source on circumference of missile (in)
ν	Poission's ratio elastic modulus for missile
$\xi(\tau)$	Abscissa of an expansion wave source on circumference of missile (in)
ρ, ρ_i	Undisturbed or initial mass density of water ($lb\text{-}sec^2/in^4$)
$\rho(\tau)$	Radius of an expansion wave from an edge source on missile face (in)
ρ_f	Final or shocked water mass density ($lb\text{-}sec^2/in^4$)
ρ_m	Undisturbed mass density of missile ($lb\text{-}sec^2/in^4$)
σ_x, σ_y	Lateral and transverse stresses in shocked missile material (psi)

NOMENCLATURE (Cont.)

σ_z	Axial stress in missile due to impact shock (psi)
τ	Time parameter which describes the expansion wave profile curve on the missile's rigid face (sec.)
ϕ	Angle between deformed missile face and the line between the expansion wave and water surface contact points in the plane of symmetry (rad.)
ψ	Angle between wetting side of missile and the line between the expansion wave and water surface contact points in the plane of symmetry (rad.)
ω	Deformation angle of missile face from its rigid profile (rad.)

APPENDIX A

'IMPACT' PROGRAM LISTING

```
 82/09/02. 16.45.22.
PRØGRAM   IMPACT

PRØGRAM IMPACT (INPUT,ØUTPUT)
CØMMØN /TABLE/ N,D
CØMMØN /PHYS/ RHØ,V1,ALPHA,RHØM,AM,EZ,ØMEGA,U,A,BETA,Q,P
REAL NU
C
C         CØNSTANT PHYSICAL DATA ENTRY
C
C    RICE & WALSH SHØCK WAVE SPEED DATA
DIMENSIØN D(2,30)
DATA N/30/
DATA (D(1,I),I=1,30)/-72.5E3,0.,72.5E3,145.0E3,217.5E3,290.0E3,362.5E3,
+435.0E3,580.0E3,725.0E3,870.0E3,1.015E6,1.160E6,1.305E6,1.450E6,
+1.595E6,1.740E6,1.885E6,2.030E6,2.175E6,2.465E6,2.755E6,2.900E6,
+3.045E6,3.335E6,3.625E6,4.350E6,5.075E6,5.800E6,6.525E6/
DATA (D(2,I),I=1,30)/25.98E3,58.39E3,79.05E3,92.60E3,103.6E3,113.0E3,
+121.4E3,128.9E3,142.2E3,153.9E3,164.3E3,173.8E3,182.6E3,190.9E3,
+198.6E3,205.8E3,212.8E3,219.3E3,225.6E3,231.6E3,242.5E3,253.1E3,
+258.5E3,263.0E3,273.3E3,281.5E3,301.9E3,320.1E3,336.7E3,352.1E3/
DATA PI/3.1416/,RHØ/93.41E-6/,C1/58.39E3/
PRINT 1
1 FØRMAT (//,'ØBLIQUE ØR NØRMAL WATER ENTRY IMPACT ØF FLAT FACED CYLINDRIC
+AL MISSLES (CØNST. ENTRY VEL.)')
C
C         ENTRY ØF INDEPENDENT VARIABLES
C
PRINT 2
2 FØRMAT (/,'ENTER RADIUS ØF MISSILE, IN =')
READ *, R
PRINT 3
3 FØRMAT (/,'IMPACT VELØCITY ØF MISSILE, FT/SEC. =')
READ *, V2
V1=12.*V2
PRINT 4
4 FØRMAT (/,'ANGLE ØF ENTRY FRØM VERTICAL, DEG. =')
READ *, ALPHA1
ALPHA=ALPHA1*PI/180.
PRINT 7
7 FØRMAT (/,'ENTER WEIGHT DENSITY ØF MISSILE, LB/IN**3 =')
```

```
READ *, RHØM1
RHØM=RHØM1/386.07
PRINT 8
8 FØRMAT (/,'YØUNGS ELASTIC MØDULUS, PSI =')
READ *, E
PRINT 9
9 FØRMAT (/,'PØISSØNS RATIØ FØR MISSILE =')
READ *, NU
PRINT 11
11 FØRMAT (/,'ENTER THE NUMBER ØF TIME INTERVALS DESIRED =')
READ *, M
C
C        ØUTPUT TABLE HEADER
C
PRINT 12
12 FØRMAT (//,17X,'WATER ENTRY IMPACT ØF FLAT FACED CYLINDRICAL MISSILES (
+CØNST. ENTRY VEL.)',/,43X,'SHØCK DYNAMICS RESPØNSE')
PRINT 13, R,RHØM1,ALPHA1,E,V2,NU
13 FØRMAT (/,7X,'MISSILE RADIUS = ',F6.3,' IN',13X,'WEIGHT DENSITY = ',
+F5.3,' LB/IN**3',/,7X,'ENTRY ANGLE = ',F6.3,' DEG.',14X,'YØUNGS MØDULUS =
+',E9.3,' PSI',/,7X,'ENTRY VELØCITY = ',F5.0,' FT/SEC.',9X,'PØISSØNS RATIØ
+= ',F5.3)
C
C        CØMPUTATIØN ØF CØNSTANT SHØCK DYNAMICS RESULTS
C
C    ELASTIC WAVE SPEED IN MISSILE
AM=SQRT(E*(1.-NU)/(RHØM*(1.+NU)*(1.-2.*NU)))
C
C    SHØCK PRESSURE ITERATIVE CØMPUTATIØN
C    INITIAL TRIAL SHØCK PRESSURE, P1
J=1
P1=RHØ*C1*V1
CALL PRESS(P1,G1)
P2=P1+0.1*P1
15 CALL PRESS(P2,G2)
CALL SECANT (0.0001,30,J,P1,G1,P2,G2)
IF (J.GT.0) GØ TØ 15
C
C    CØNVERGENT VALUE ØF SHØCK PRESSURE TØ +,- 0.01 PER CENT, P
CALL PRESS(P2,G)
C
C    EXPANSIØN WAVE SPEED ACRØSS FACE ØF MISSILE, C
DELTA=PI/2.+ØMEGA
PHI=ASIN((V1-EZ*AM)*SIN(DELTA)/C1)
PSI=PI-(DELTA+PHI)
C=C1*SIN(PSI)/SIN(DELTA)*CØS(ØMEGA)
C
C    TIME AT FULL WETTING ENTRY
TE=2.*R*TAN(ALPHA)/V1
C
C    TIME DURATIØN ØF IMPACT SHØCK
TI=R*TAN(ALPHA)/V1+R/C
```

```
C
C     PRINT ØUT ØF CØNSTANT SHØCK DYNAMICS RESULTS
 ØMEGA1=ØMEGA*180./PI
 BETA1=BETA*180./PI
 PRINT 5, AM,EZ,ØMEGA1,U,A,BETA1,Q,P,C,TE,TI
5 FØRMAT (/,'SHØCK WAVE SPEED IN MISSILE =',E10.4,' IN/SEC.',/,
+'ELASTIC STRAIN ØN MISSILE FACE =',E10.4,' IN/IN',/,
+'DEFØRMATIØN ANGLE ØF MISSILE FACE =',E10.4,' DEG.',/,
+'WATER PARTICLE VELØCITY IN SHØCK =',E10.4,' IN/SEC.',/,
+'SHØCK WAVE SPEED IN WATER =',E10.4,' IN/SEC.',/,
+'MACH ANGLE ØF SHØCK WAVE IN WATER =',E10.4,' DEG.',/,
+'WATER PARTICLE SPEED NØRMAL TØ SHØCK WAVE =',E10.4,' IN/SEC.',/,
+'RANKINE-HUGØNIØT SHØCK PRESSURE =',E10.4,' PSI',/,
+'EXPANSIØN WAVE SPEED ACRØSS MISSILE FACE =',E10.4,' IN/SEC.',/,
+'TIME AT FULL WETTING ENTRY =',E10.4,' SEC.',/,
+'TIME DURATIØN ØF IMPACT SHØCK =',E10.4,' SEC.')
C
C        CØMPUTATIØN ØF TIME DEPENDENT RESULTS
C
 PRINT 6
6 FØRMAT (/,6X,'TIME',10X,'SHØCK AREA',4X,'SHØCK FØRCE',3X,
+'IMPULSE',/,6X,'SEC.',10X,'IN**2',9X,'LB',12X,'LB-SEC.',/)
C
C     TIME STEP INTERVAL, DELTA T
 H=TI/(2.*M)
C
C     INITIALIZE IMPULSE
 FT=0.
 L=2*M+1
 DØ 16 I=1,L
 T=(I-1)*H
C
C     SHØCK PRESSURE AREA DETERMINATIØN, S
 IF (ALPHA.GT.0.) GØ TØ 18
 S=PI*(R-C*T)**2
 GØ TØ 19
C
C     WETTING LINE SPEED
18 V=V1/TAN(ALPHA)
C
C     WETTING LINE SPEED CHECK
 IF (V.GT.C) GØ TØ 10
 PRINT 20, V,C
20 FØRMAT (//,'THE WETTING LINE SPEED',E10.4,' IN/SEC.','IS LESS THAN THE
+EXPANSIØN WAVE SPEED',E10.4,' IN/SEC.',/,'NØ IMPACT SHØCK ØCCURS')
 STØP
10 CALL AREA (R,V,C,T,200,S1)
 S=2.*S1
C
C     IMPACT SHØCK FØRCE
19 F=P*S
C
C     INTEGRATE FØRCE TØ IMPULSE
 CALL SIMP (I,H,F,FT)
```

```
C
C    PRINT ØUT ØF SHØCK DYNAMICS RESULTS
 IF (I.EQ.2*(I/2)) GØ TØ 16
 PRINT 17, T,S,F,FT
 16 CØNTINUE
 17 FØRMAT (4(4X,E10.4))
 END
C
C         RANKINE-HUGØNIØT SHØCK PRESSURE CØMPUTATIØN
C
 SUBRØUTINE PRESS (P,G)
 CØMMØN /PHYS/ RHØ,V1,ALPHA,RHØM,AM,EZ,ØMEGA,U,A,BETA,Q,PS
C
C    AXIAL STRAIN IN MISSILE UNDER SHØCK PRESSURE
 EZ=P/(RHØM*AM*AM)
C
C    DEFØRMATIØN ANGLE ØF MISSILE FACE FRØM RIGID FACE
 ØMEGA=ATAN(EZ*AM*TAN(ALPHA)/V1)
C
C    WATER PARTICLE VELØCITY
 U=(V1-EZ*AM)*CØS(ØMEGA)
C
C    WATER SHØCK SPEED FRØM TRIAL PRESSURE (RICE & WALSH)
 CALL ITABLE (P,A)
C
C    SURFACE DISTURBANCE SPEED CHECK
 IF (V1/A.GT.SIN(ALPHA)) GØ TØ 2
 DS=V1/SIN(ALPHA)
 PRINT 1, DS,A
 1 FØRMAT (//,'SURFACE DISTURBANCE SPEED',E10.4,' IN/SEC. IS LESS THAN THE
+SHØCK WAVE SPEED',E10.4,' IN/SEC.',/,'NØ IMPACT SHØCK ØCCURS.')
 STØP
C
C    WATER SHØCK MACH ANGLE
 2 BETA=ASIN(A*SIN(ALPHA)/V1)
C
C    CØMPØNENT ØF WATER PARTICLE VELØCITY NØRMAL TØ SHØCK FRØNT
 Q=U*CØS(BETA-(ALPHA-ØMEGA))
C
C    RANKINE-HUGØNIØT SHØCK PRESSURE: ØBLIQUE SHØCKS
 PS=RHØ*A*Q
C
C    SHØCK PRESSURE ERRØR FUNCTIØN
  G=P-PS
 RETURN
 END
C
C         4 PØINT LAGRANGIAN INTERPØLATIØN ØF THE
C         RICE & WALSH SHØCK PRESSURE & WAVE SPEED DATA TABLE
C
 SUBRØUTINE ITABLE (X,Y)
 CØMMØN /TABLE/ N,T
 DIMENSIØN T(2,1)
```

```
M=N-2
DØ 1 I=2,M
IF (X.GT.T(1,I).AND.X.LE.T(1,I+1)) GØ TØ 3
1 CØNTINUE
PRINT 2, X
2 FØRMAT (/,'X =',E10.4,' LIES ØUTSIDE THE INNER RANGE ØF THE TABLE')
STØP
3 X1=T(1,I-1)
Y1=T(2,I-1)
X2=T(1,I)
Y2=T(2,I)
X3=T(1,I+1)
Y3=T(2,I+1)
X4=T(1,I+2)
Y4=T(2,I+2)
D1=(X1-X2)*(X1-X3)*(X1-X4)
D2=(X2-X1)*(X2-X3)*(X2-X4)
D3=(X3-X1)*(X3-X2)*(X3-X4)
D4=(X4-X1)*(X4-X2)*(X4-X3)
Y=(X-X2)*(X-X3)*(X-X4)*Y1/D1
Y=Y+(X-X1)*(X-X3)*(X-X4)*Y2/D2
Y=Y+(X-X1)*(X-X2)*(X-X4)*Y3/D3
Y=Y+(X-X1)*(X-X2)*(X-X3)*Y4/D4
RETURN
END
C
C         SECANT ITERATIØN ØF SHØCK PRESSURE TØ YIELD A RØØT
C         ØF THE PRESSURE ERRØR FUNCTIØN G(P)=0.
C
SUBRØUTINE SECANT(E,N,I,X1,F1,X2,F2)
IF (ABS(F2-F1).EQ.0.) GØ TØ 1
X3=X2-(X2-X1)*F2/(F2-F1)
X1=X2
F1=F2
X2=X3
I=I+1
1 IF (ABS((X2-X1)/X1).LE.E) GØ TØ 2
IF (I.GT.N) GØ TØ 3
RETURN
2 I=-1
RETURN
3 PRINT 4, N,X1,F1,X2,F2
4 FØRMAT (/,'SECANT METHØD DØES NØT CØNVERGE FØR N =',I3,/,
+'X1 =',E12.6,4X,'F1 =',E12.6,/'X2 =',E12.6,4X,'F2 =',E12.6)
STØP
END
C
C         HALF AREA BETWEEN WETTING LINE & EXPANSIØN WAVE FRØNT PRØFILE
C
SUBRØUTINE AREA (R,V,C,T,N,A)
A=0.
IF (T.LE.0..ØR.T.GE.R/V+R/C-1E-9) GØ TØ 4
```

```
X1=0.
X2=0.
Y1=0.
Y2=0.
T1=T
IF (T.GT.2.*R/V) T1=2.*R/V
M=N+1
DØ 1 I=1,M
C
C    TIME PARAMETER FØR PRØFILE CURVE, 0<TAU<T1
TAU=(I-1)*(T1/N)
XI=R-V*TAU
ETA=SQRT(R*R-XI*XI)
RHØ=C*(T-TAU)
DELTA=(C/V)*RHØ
RT=SQRT(R*R*RHØ*RHØ-ETA*ETA*DELTA*DELTA)
C
C    CØØRDINATES ØF PRØFILE CURVE: X(TAU),Y(TAU)
X=XI-(ETA*ETA*DELTA+XI*RT)/(R*R)
Y=ETA*(X+DELTA)/XI
C
C    SIMPSØNS RULE INTEGRATIØN FØR UNEQUAL INTERVALS ØN X
IF (I.GT.1) GØ TØ 2
X1=X
Y1=Y
GØ TØ 1
2 IF (I.GT.2*(I/2)) GØ TØ 3
X2=X
Y2=Y
GØ TØ 1
3 P1=(X1**3-X**3)/3.
P2=(X1*X1-X*X)/2.
P3=X1-X
A=A+(P1-(X2+X)*P2+X2*X*P3)*Y1/((X1-X2)*(X1-X))
A=A+(P1-(X1+X)*P2+X1*X*P3)*Y2/((X2-X1)*(X2-X))
A=A+(P1-(X1+X2)*P2+X1*X2*P3)*Y/((X-X1)*(X-X2))
X1=X
Y1=Y
1 CØNTINUE
4 RETURN
END
C
C          SIMPSØN'S 1/3 RULE INTEGRATIØN
C
SUBRØUTINE SIMP (I,H,Y,A)
IF (I.GT.1) GØ TØ 1
Y1=Y
RETURN
1 IF (I.GT.2*(I/2)) GØ TØ 2
Y2=Y
RETURN
2 A=A+(Y1+4.*Y2+Y)*H/3.
Y1=Y
RETURN
END
```

APPENDIX B

LISTING OF EXAMPLE OUTPUT

```
run

 82/09/02. 16.52.03.
PRØGRAM   IMPACT


ØBLIQUE ØR NØRMAL WATER ENTRY IMPACT ØF FLAT FACED CYLINDRICAL MISSLES (CØNST. ENTRY VEL.)

ENTER RADIUS ØF MISSILE, IN =
? 6

IMPACT VELØCITY ØF MISSILE, FT/SEC. =
? 1500

ANGLE ØF ENTRY FRØM VERTICAL, DEG. =
? 5

ENTER WEIGHT DENSITY ØF MISSILE, LB/IN**3 =
? 0.1

YØUNGS ELASTIC MØDULUS, PSI =
? 10e6

PØISSØNS RATIØ FØR MISSILE =
? 0.333

ENTER THE NUMBER ØF TIME INTERVALS DESIRED =
? 50


                 WATER ENTRY IMPACT ØF FLAT FACED CYLINDRICAL MISSILES (CØNST. ENTRY VEL.)
                                         SHØCK DYNAMICS RESPØNSE

       MISSILE RADIUS =  6.000 IN            WEIGHT DENSITY =  .100 LB/IN**3
       ENTRY ANGLE =  5.000 DEG.             YØUNGS MØDULUS =  .100E+08 PSI
       ENTRY VELØCITY = 1500. FT/SEC.        PØISSØNS RATIØ =  .333

SHØCK WAVE SPEED IN MISSILE = .2405E+06 IN/SEC.
ELASTIC STRAIN ØN MISSILE FACE = .8301E-02 IN/IN
DEFØRMATIØN ANGLE ØF MISSILE FACE = .5560E+00 DEG.
WATER PARTICLE VELØCITY IN SHØCK = .1600E+05 IN/SEC.
SHØCK WAVE SPEED IN WATER = .8920E+05 IN/SEC.
MACH ANGLE ØF SHØCK WAVE IN WATER = .2559E+02 DEG.
```

WATER PARTICLE SPEED NØRMAL TØ SHØCK WAVE = .1493E+05 IN/SEC.
RANKINE-HUGØNIØT SHØCK PRESSURE = .1244E+06 PSI
EXPANSIØN WAVE SPEED ACRØSS MISSILE FACE = .5600E+05 IN/SEC.
TIME AT FULL WETTING ENTRY = .5833E-04 SEC.
TIME DURATIØN ØF IMPACT SHØCK = .1363E-03 SEC.

TIME SEC.	SHØCK AREA IN**2	SHØCK FØRCE LB	IMPULSE LB-SEC.
0.	0.	0.	0.
.2726E-05	.1387E+01	.1725E+06	.1902E+00
.5453E-05	.3850E+01	.4788E+06	.1057E+01
.8179E-05	.6935E+01	.8625E+06	.2870E+01
.1091E-04	.1046E+02	.1301E+07	.5809E+01
.1363E-04	.1431E+02	.1780E+07	.1000E+02
.1636E-04	.1840E+02	.2289E+07	.1554E+02
.1908E-04	.2265E+02	.2817E+07	.2250E+02
.2181E-04	.2701E+02	.3359E+07	.3092E+02
.2454E-04	.3140E+02	.3905E+07	.4082E+02
.2726E-04	.3578E+02	.4450E+07	.5221E+02
.2999E-04	.4010E+02	.4987E+07	.6507E+02
.3272E-04	.4429E+02	.5509E+07	.7938E+02
.3544E-04	.4831E+02	.6008E+07	.9509E+02
.3817E-04	.5209E+02	.6478E+07	.1121E+03
.4089E-04	.5557E+02	.6910E+07	.1304E+03
.4362E-04	.5866E+02	.7296E+07	.1498E+03
.4635E-04	.6130E+02	.7623E+07	.1701E+03
.4907E-04	.6335E+02	.7879E+07	.1913E+03
.5180E-04	.6468E+02	.8044E+07	.2130E+03
.5453E-04	.6506E+02	.8092E+07	.2350E+03
.5725E-04	.6405E+02	.7965E+07	.2569E+03
.5998E-04	.6043E+02	.7516E+07	.2781E+03
.6270E-04	.5631E+02	.7002E+07	.2979E+03
.6543E-04	.5232E+02	.6507E+07	.3163E+03
.6816E-04	.4847E+02	.6028E+07	.3334E+03
.7088E-04	.4478E+02	.5569E+07	.3492E+03
.7361E-04	.4122E+02	.5126E+07	.3638E+03
.7634E-04	.3782E+02	.4704E+07	.3772E+03
.7906E-04	.3457E+02	.4299E+07	.3894E+03
.8179E-04	.3146E+02	.3913E+07	.4006E+03
.8451E-04	.2850E+02	.3545E+07	.4108E+03
.8724E-04	.2568E+02	.3193E+07	.4200E+03
.8997E-04	.2302E+02	.2863E+07	.4282E+03
.9269E-04	.2048E+02	.2546E+07	.4356E+03
.9542E-04	.1813E+02	.2255E+07	.4421E+03
.9815E-04	.1588E+02	.1975E+07	.4479E+03
.1009E-03	.1380E+02	.1716E+07	.4529E+03
.1036E-03	.1186E+02	.1475E+07	.4573E+03
.1063E-03	.1005E+02	.1250E+07	.4610E+03
.1091E-03	.8417E+01	.1047E+07	.4641E+03
.1118E-03	.6916E+01	.8601E+06	.4667E+03
.1145E-03	.5551E+01	.6904E+06	.4688E+03

```
.1172E-03   .4343E+01   .5401E+06   .4705E+03
.1200E-03   .3288E+01   .4088E+06   .4718E+03
.1227E-03   .2364E+01   .2941E+06   .4727E+03
.1254E-03   .1594E+01   .1982E+06   .4734E+03
.1281E-03   .9692E+00   .1205E+06   .4738E+03
.1309E-03   .4895E+00   .6087E+05   .4741E+03
.1336E-03   .1572E+00   .1955E+05   .4742E+03
.1363E-03  0.          0.           .4742E+03

SRU     10.185 UNTS.

RUN COMPLETE.
bye

249      LOG OFF    16.55.32.
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```

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